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Research Article

An Intelligent and Robust Framework towards Anomaly Detection, Medical Diagnosis, and Shortest Path Problems Based on Interval-Valued T-Spherical Fuzzy Information

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The recent emerging advancements in the domain of the fuzzy sets are the framework of the T-spherical fuzzy set (TSFS) and interval valued T-spherical fuzzy set (IVTSFS). Keeping in view the promising significance of the latest research trend in the fuzzy sets and the enabling impact of IVTSFS, we proposed a novel framework for decision assembly using interval valued TSFS based upon encompassing the four impressive dimensions of human judgement including favor, abstinence, disfavor, and refusal degree. Another remarkable contribution is the optimization of information modeling and prevention of information loss by redefining the concept of each membership in interval. Moreover, the proposed research made a worthy contribution work by demonstrating the effective utilization of the interval valued TSFS based framework in anomaly detection, medical diagnosis, and shortest path problem. The proposed work demonstrates the effective remedial measure for the anomaly detection problem based on several parameters using the aggregation operators of IVTSFS. Moreover, the interval valued T-spherical fuzzy relations and their composition are illustrated to investigate the medical diagnosis problem. Furthermore, the notion of interval valued T-spherical fuzzy graph is also presented and fundamental notions of graph theory are also demonstrated with the help of real world instances. In the context of interval valued T-spherical fuzzy graphs (IVTSFGs), a modified Dijkstra Algorithm (DA) is developed and applied to the shortest path problem. The in-depth quantitative assessment and comparative analysis revealed that the proposed notion outpaces contemporary progressive approaches.

1. Introduction

Zadeh [1] developed a novel way of describing an entity's membership to a set using the idea of a fuzzy set (FS) which was further enhanced by Atanassov [2] who proposed the perception of an intuitionistic fuzzy set (IFS). In an IFS, the relation of an object to a set is described by the membership as well as nonmembership grades (NMGs) denoted by ş and d on the unit interval in a way that $0 \leq \text{sum}(ş, d) \leq 1$. The condition $0 \leq \text{sum}(ş, d) \leq 1$ on IFS makes it less effective in some cases because it does not allow us to choose the values of ş and d independently. This issue was addressed by the framework of the Pythagorean fuzzy set (PyFS) developed with a relatively stronger condition, i.e., $0 \leq \text{sum}(ş^2, d^2) \leq 1$. The concept of PyFS also becomes limited in some cases and a new generalized ortho pair FS known as q-rung ortho pair FS (Q-ROPFS) introduced by Yager [3] with a condition that $0 \leq \text{sum}(ş^q, d^q) \leq 1$ such that $q \geq 1$. The comparative evaluation of the spaces of IFS, PyFS, and Q-ROPFS is described in Figure 1. In Figure 1, the turquoise color stripe models the aggregated perimeter of IFSs, the light blue semicircle represents PyFS, and the red curve illustrates Q-ROPFS for

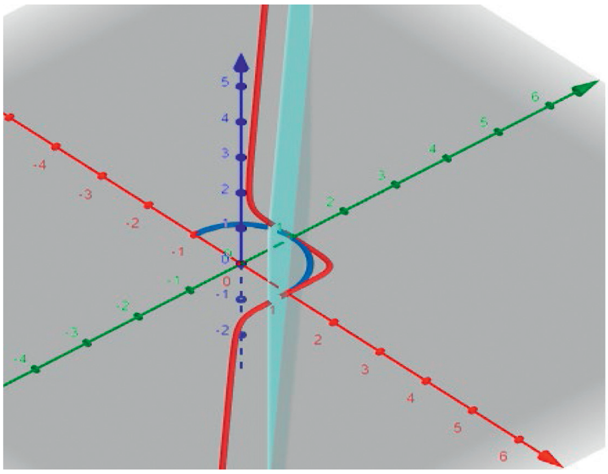

Figure 1: Comparison of spaces of intuitionistic fuzzy set, Pythagorean fuzzy set, and q-rung ortho pair fuzzy set.

$q = 7$. IFS, PyFS, and Q-ROPFS are only effective in situations with discrete resolutions, i.e., yes or no.

Cuong [4, 5] proposed a new theory of picture fuzzy set (PFS) which described four types of grades ş, ı, d, and R denoting membership, nonmembership, abstinence, and refusal degree such that $0 \leq \mathrm{sum}(ş, ı, d) \leq 1$ and $R = 1 - \mathrm{sum}(ş, ı, d)$. Cuong's idea of PFS generalizes the theory of IFS but constrained by the condition $0 \leq \mathrm{sum}(ş, ı, d) \leq 1$ due to which grades could not be assigned independently. Therefore, a novel idea of TSFS and SFS generalizing IFS, PyFS, Q-ROPFS, and PFS with a condition on membership grade (MG) $0 \leq \mathrm{sum}(ş^n, ı^n, d^n) \leq 1$ and $R = \sqrt[n]{\mathrm{sum}(ş^n, ı^n, d^n)}$ such that $n \in \mathbb{Z}^+$ and $n \geq 1$ was proposed by Mahmood [6]. The comparison of spaces of PFSs and SFSs is described in Figure 2 where the pink line represents the boundary space of PFSs, and the dark grey curves represent the boundary space of SFSs. One may also refer to [6, 7] for further knowledge of PFSs, SFSs, and TSFSs.

In real-life problems, the MG of an object could not be assigned due to some type of uncertainty, and hence, [8] proposed IIVFS with the definition of entity's membership with closed subinterval $(ş^L, ş^U)$ of the unit interval. An IVFS easily reduces to FS if we take $ş^L = ş^U$. Following this phenomenon, the same type of structures has been proposed for IFSs, PyFSs, and PFSs. The concept of interval valued IFS (IVIFS) is proposed in [9], interval valued PyFS (IVPyFS) is developed in [10], and interval valued PFS (IVPFS) is proposed in [5].

The concepts of IVIFS, IVPyFS, and IVPFS are reduced to IFS, PyFS, and PFS. Keeping the importance of these concepts, Ullah [11] proposed the framework of IVTSFS. Ullah [11] also described the importance of describing membership degrees. All these concepts have been applied to various practical problems, such as [12, 13] which described MADM problems using some normal interactive aggregation operators and VIKOR method based on IFSs, [14] which is based on multiple parameters based two-person zero sum game with IF information, and [15–22] which are dealing with MADM using different approaches.

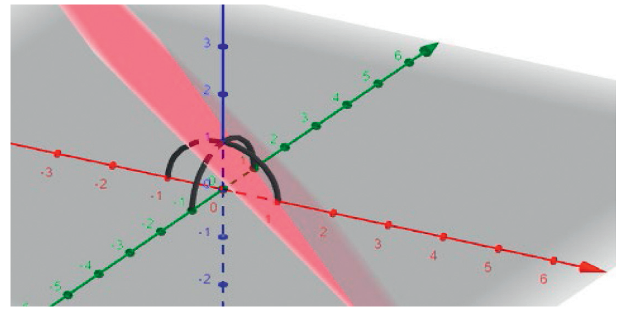

Figure 2: Comparison of spaces of PFS and SFS.

The concept of fuzzy graph (FG) is developed by [23] as a generalized crisp graph and comprehensively examined by Rosenfeld [24]. The idea of FG is indeed very helpful in various problems including classification, clustering, anomaly/malware detection, shortest path problems, networking problems, slicing, and in the description of group structures, etc. Due to the uncertainties lying in these problems, FG was further generalized to the concept of an intuitionistic fuzzy graph (IFG) where nodes and edges are assumed to be intuitionistic fuzzy numbers (IFNs) in [25]. The graphical aspects of PyFS are discussed in [25], the notion of PFG is developed in [26], and its operations are studied. Reference [27] developed the theory of IVFG and [28] is based on IVIFG. The TSFG's notion is also proposed in [29] which provided a more flexible ground for dealing with imprecise information. Some quality work on these graphs can be found in [30–34].

Malware/Anomaly detection is a critical cyber world problem, and it is crucial with malware's prevalence because it functions as threat intelligence and detection system against anomaly based cyber-attacks. In anomaly detection, we establish a process using aggregation operators regarding the classification of a given sample of Windows based Portable Executable (PE) file. The aggregation operators are applied to the decision matrix containing the attributes of the given sample to calculate the score values. The score value of each sample file is used by the problem solver to determine the nature of the portable executable file, i.e., either anomaly (malicious) or benign (nonmalicious). Some novel research work in the subject domain has been so far addressed in [35–40]. Medical diagnosis is also another important practical problem that has been widely discussed in the context of FSs and its algebraic extensions. In medical diagnosis, we establish some relations between patients, symptoms and symptoms, diagnosis. Then by the composition of the previous two relations, we determine the diagnosis for patients. This type of phenomenon has been so far discussed in [6, 41–43]. Furthermore, the SP problem is also one of the interesting problems that have much impotence in computer sciences and fuzzy algebra. In this phenomenon, we find the SP in a network of nodes and edges from the source node to the destination node. Several algorithms have been developed for finding SP problems. Some notable work in this regard could be found in [44–48].

The problem of bounding the membership's aggregate, abstinence, and NMGs between one and zero is also experienced by Cuong's PFS. The problem also revokes the privilege from the problem solvers to opt three aforementioned characteristic functions. Considering the problem, Mahmood [49] introduced a novel framework of the SFS. There is a dire need to develop innovative models keeping in view several characteristic functions with no shortcomings [53, 254]. The significance of representing the MGs in terms of intervals strongly motivated us for our proposed research work. The research work proposed the IVTSFS with WA and WG aggregation operators and demonstration of its effective application for MADM. The notable benefactions of this manuscript are as follows:

(1) To propose the novel concept of the IVTSFS that encompasses an event covering MGs in terms of intervals without any constraint
(2) To propose novel aggregation tools and its effective application in those specialized domains which remain a problem for the application of IVFS, IVIFS, and IVPFS
(3) To explore the problem of MADM and utilize the aggregation operators of IVTSFSs for decision making in real world scenarios

The extensive model evaluation followed by indepth comparative assessment has demonstrated that the proposed aggregation operators with appropriate constraints outperform the preexisting aggregation operators. In this manuscript, we developed a new concept of interval valued T-spherical fuzzy graphs (IVTSFGs). The concepts of IVTSFSs and IVTSFGs are applied to the problem of anomaly detection, medical diagnosis, and SP problem. Also, it is proved that none of the existing tools could handle the data provided in IVTSFS and on the other hand, it is also proved that the structure of IVTSFSs and IVTSFG process the problem handsomely in lieu of FS or FG, IFS or IFG, IVIFS or IVIFG, IVPFS, SFS, and TSFS.

This manuscript is arranged as follows: Section 1 described a brief history of existing work and motivation for new work, followed by a review of fundamental definitions in Section 2. Sections 3 and 4 covered the concept of IVTSFSs and its significance, respectively. Section 5 discussed the notion of IVTSFG and its some graphical notions. Section 6 discussed three applications of IVTSFSs and IVTSFGs including anomaly detection, medical diagnosis, and shortest path based on interval valued T-spherical fuzzy relations (IVTSFRs). Section 7 narrates the concluding remarks followed by the future direction.

2. Basic Concepts

In our study, we denote a universal set by X. Further, by $\c{s}, \imath, \mathrm{d}$, and R we denote membership, abstinence, nonmembership, and refusal functions maps on $[0,1]$ interval.

Definition 1 (see [2]). An IFS is expressed as $A = \{\varkappa, (\c{s}(\varkappa), d(\varkappa))\}$ such that $0 \le \mathrm{sum}(\c{s}, d) \le 1$ and $R = 1 - \mathrm{sum}(\c{s}, d)$ are known as hesitancy index.

Definition 2 (see [4]). A PFS is expressed as $A = \{\varkappa, (\c{s}(\varkappa), \imath(\varkappa), \mathrm{d}(\varkappa))\}$ such that $0 \le \mathrm{sum}(\c{s}, \imath, \mathrm{d}) \le 1$ and $R = 1 - \mathrm{sum}(\c{s}, \imath, \mathrm{d})$ are termed as refusal degree.

Definition 3 (see [6]). A SFS is expressed as $A = \{\varkappa, (\c{s}(\varkappa), \imath(\varkappa), \mathrm{d}(\varkappa))\}$ such that $0 \le \mathrm{sum}(\c{s}^2, \imath^2, d^2) \le 1$ and $R = \sqrt{\mathrm{sum}(\c{s}^2, \imath^2, d^2)}$ are termed as refusal degree.

Definition 4 (see [6]). A TSFS is expressed as $A = \{\varkappa, (\c{s}(\varkappa), \imath(\varkappa), \mathrm{d}(\varkappa))\}$ such that $0 \le \mathrm{sum}(\c{s}^n, \imath^n, d^n) \le 1$ for $n \ge 1, n \in \mathbb{Z}^+$ and $R = \sqrt[n]{\mathrm{sum}(\c{s}^n, \imath^n, d^n)}$ are termed as refusal degree.

The triplet $(\c{s}, \imath, \mathrm{d})$ is a picture fuzzy and spherical fuzzy and T-spherical fuzzy numbers, respectively, for Definitions 2–4.

Definition 5 (see [9]). An IVIFS is expressed as $A = \{\varkappa, (\c{s}(\varkappa), d(\varkappa))\}$ such that $\c{s} = [\c{s}^L. \c{s}^U]$ and $d = [d^L, d^U]$ provided that $0 \le \mathrm{sum}(\c{s}^U, d^U) \le 1$ and $R = [R^L, R^U] = [1 - \mathrm{sum}(\c{s}^U, d^U), 1 - \mathrm{sum}(\c{s}^L, d^L)]$ are known as hesitancy index. Further, the duplet $([\c{s}^L. \c{s}^U], [d^L, d^U])$ is an IVIFN.

Definition 6 (see [5]). An IVPFS is expressed as $A = \{\varkappa, (\c{s}(\varkappa), \imath(\varkappa), \mathrm{d}(\varkappa))\}$ such that $\c{s} = [\c{s}^L. \c{s}^U], \imath = [\imath^L, \imath^U]$ and $d = [d^L, d^U]$ provided that $0 \le \mathrm{sum}(\c{s}^U, \imath^U, d^U) \le 1$ and $R = [R^L, R^U] = [1 - \mathrm{sum}(\c{s}^U, \imath^U, d^U), 1 - \mathrm{sum}(\c{s}^L, \imath^L, d^L)]$ are known as refusal degree. Further, the triplet $([\c{s}^L. \c{s}^U], [\imath^L, \imath^U], [d^L, d^U])$ is an IVPFN.

Example 1. Figure 3 corresponds to FG, while Figure 4 to its complement.

Definition 7 (see [24]). An IFG is defined by $G = (V, E)$ V and E are the collections nodes and edges, respectively.

(1) For all $v \in V$ is categorized by $\c{s}$ and d denoting the MG and NMG of $v \in V$ s.t $0 \le \c{s} + d \le 1$. The term R denotes the hesitancy level of $v \in V$ such that $R = 1 - \c{s} - d$.
(2) Every $e \in E$ is attributed $\hat{s}$ and $Đ$ denoting the MG and the NMG defined as follows:

$$\begin{aligned}\hat{S}(v_i, v_j) &\le \min(\c{s}(v_i), \c{s}(v_j)),\\ Đ(v_i, v_j) &\le \max(d(v_i), d(v_j)),\end{aligned} \tag{1}$$

s.t. $0 \le \hat{s} + Đ \le 1$. $\check{R}$ denotes the hesitancy degree of $e \in E$ such that $\check{R} = 1 - \hat{S} - Đ$.

Example 2. Figure 5 represents IFG.

3. Interval-Valued T-Spherical Fuzzy Sets

The recently introduced concept of IVTSFSs with properties is discussed and the notion of IVTSF relation is also presented with results and discussion.

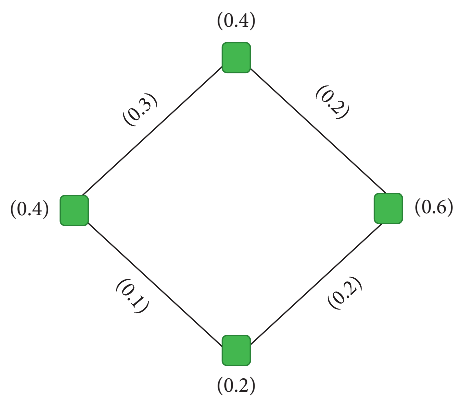

Figure 3: Fuzzy graph w.r.t Definition 7.

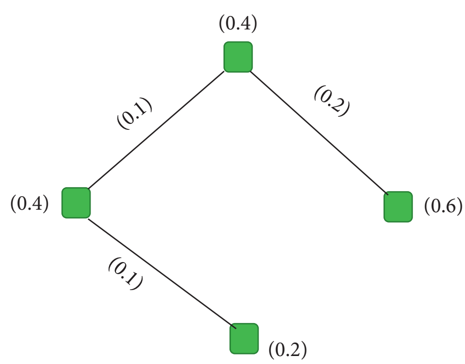

Figure 4: Complement of fuzzy graph.

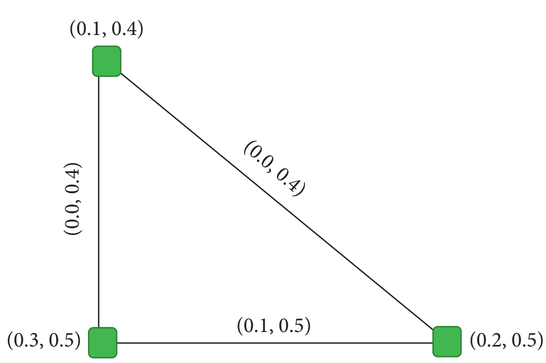

Figure 5: Intuitionistic fuzzy graph compatible with Definition 7.

Definition 8 (see [11]). An IVTSFS $A = \{\varkappa, (ş(\varkappa), ı(\varkappa), d(\varkappa))\}$ where ş, ı and d are from X to subinterval of $[0, 1]$, i.e., $ş(\varkappa) = [ş^L(\varkappa), ş^U(\varkappa)], ı(\varkappa) = [ı^L(\varkappa), ı^U(\varkappa)]$ and $d(\varkappa) = [d^L(\varkappa), d^U(\varkappa)]$ with a condition $0 \leq (ş^U(\varkappa))^n + (ı^U(\varkappa))^n + (d^U(\varkappa))^n \leq 1$ and $n \in \mathbb{Z}^+$. Moreover, the interval $R(\varkappa) = [R^U(\varkappa), R^L(\varkappa)]$ represents the level of refusal $\varkappa \in X$, i.e., $R^U(\varkappa) = \sqrt[n]{1 - (ş^L(\varkappa))^n - (ı^L(\varkappa))^n - (d^L(\varkappa))^n}$ and $R^L(\varkappa) = \sqrt[n]{1 - (ş^U(\varkappa))^n - (ı^U(\varkappa))^n - (d^U(\varkappa))^n}$.

The following theorem proves the generalization of IVTSFSs over TSFSs, IVPFSs, PFSs, IVIFSs. IFSs, IVFSs, and FSs.

Theorem 1. *In Definition 8 of IVTSFSs, if we*

(1) *Taking $ş^L = ş^U, ı^L = ı^U$ and $d^L = d^U$, then it reduces to TSFSs.*

(2) *Taking $n = 2$, then it reduces to the definition of IVSFS.*

(3) *Taking $n = 2$, $ş^L = ş^U, ı^L = ı^U$, and $d^L = d^U$, then it reduces to the definition of SFS.*

(4) *Taking $n = 1$, then it reduces to the definition of IVPFS.*

(5) *Taking $n = 1$, $ş^L = ş^U, ı^L = ı^U$ and $d^L = d^U$, then it reduces to the definition of PFS.*

(6) *Taking $n = 2$ and $ı^L = ı^U = 0$, then it reduces to the definition of IVPyFS.*

(7) *Taking $n = 2$, $ş^L = ş^U$, $ı^L = ı^U = 0$ and $d^L = d^U$, then it reduces to the definition of PyFS.*

(8) *Taking $n = 1$ and $ı^L = ı^U = 0$, then it reduces to the definition of IVIFS.*

(9) *Taking $n = 1$, $ş^L = ş^U, ı^L = ı^U = 0$ and $d^L = d^U$, then it reduces to the definition of IFS.*

(10) *Taking $n = 1$ and $ı^L = ı^U = d^L = d^U = 0$, then it reduces to the definition of IVFS.*

(11) *Taking $n = 1$ and $ş^L = ş^U, ı^L = ı^U = 0 = d^L = d^U$, then it reduces to the definition of FS.*

Definition 9 For IVITSFSs $A = \{\varkappa, (ş_A(\varkappa), ı_A(\varkappa), d_A(\varkappa))\}$ and $B = \{\varkappa, (ş_B(\varkappa), ı_B(\varkappa), d_B(\varkappa))\}$:

(1) $A \subseteq B$ if $ş_A(\varkappa) \leq ş_B(\varkappa)$, $ı_A(\varkappa) \leq ı_B(\varkappa)$ and $d_A(\varkappa) \geq d_B(\varkappa)$, where

$$\begin{aligned} ş_A(\varkappa) \leq ş_B(\varkappa) &\Longrightarrow ş_A^L(\varkappa) \leq ş_B^L(\varkappa) \text{ and } ş_A^U(\varkappa) \leq ş_B^U(\varkappa), \\ ı_A(\varkappa) \leq ı_B(\varkappa) &\Longrightarrow ı_A^L(\varkappa) \leq ı_B^L(\varkappa) \text{ and } ı_A^U(\varkappa) \leq ı_B^U(\varkappa), \\ d_A(\varkappa) \geq d_B(\varkappa) &\Longrightarrow d_A^L(\varkappa) \geq d_B^L(\varkappa) \text{ and } d_A^U(\varkappa) \geq d_B^U(\varkappa). \end{aligned} \quad (2)$$

(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(3) $A \cup B = \{\varkappa, (ş_A(\varkappa) \vee ş_B(\varkappa), ı_A(\varkappa) \wedge ı_B(\varkappa) \text{ and } d_A(\varkappa) \wedge d_B(\varkappa))\}$, where

$$\begin{aligned}&\S_A(\varkappa)\vee\S_B(\varkappa)\Longrightarrow\S_A^L(\varkappa)\vee\S_B^L(\varkappa)\text{ and }\S_A^U(\varkappa)\vee\S_B^U(\varkappa),\\&\imath_A(\varkappa)\wedge\imath_B(\varkappa)\Longrightarrow\imath_A^L(\varkappa)\wedge\imath_B^L(\varkappa)\text{ and }\imath_A^U(\varkappa)\wedge\S_B^U(\varkappa),\\&d_A(\varkappa)\wedge d_B(\varkappa)\Longrightarrow d_A^L(\varkappa)\wedge d_B^L(\varkappa)\text{ and }d_A^U(\varkappa)\wedge d_B^U(\varkappa).\end{aligned}\tag{3}$$

(4) $A\cap B=\{\varkappa,(\S_A(\varkappa)\wedge\S_B(\varkappa),\ \imath_A(\varkappa)\wedge\imath_B(\varkappa)\text{ and }d_A(\varkappa)\vee d_B(\varkappa))\}$ where

$$\begin{aligned}&\S_A(\varkappa)\wedge\S_B(\varkappa)\Longrightarrow\S_A^L(\varkappa)\wedge\S_B^L(\varkappa)\text{ and }\S_A^U(\varkappa)\wedge\S_B^U(\varkappa),\\&\imath_A(\varkappa)\wedge\imath_B(\varkappa)\Longrightarrow\imath_A^L(\varkappa)\wedge\imath_B^L(\varkappa)\text{ and}\imath_A^U(\varkappa)\wedge\imath_B^U(\varkappa).\end{aligned}\tag{4}$$

(5) $d_A(\varkappa)\vee d_B(\varkappa)\Longrightarrow d_A^L(\varkappa)\vee d_B^L(\varkappa)$ and $d_A^U(\varkappa)\vee d_B^U(\varkappa)$.

(6) $A^c=\{\varkappa,\ (d_A(\varkappa),\ \imath_A(\varkappa),\ \S_A(\varkappa))\}$.

The following theorem shows that IVTSFSs preserve the basic set theoretic properties.

Theorem 2. *For three IVTSFSs A, B and C:*

(1) The transitive law w.r.t. inclusion holds true

(2) The union and intersection are commutative

(3) The union and intersection are associative

(4) Distributive laws of the union over intersection and intersection over union hold true

(5) DE Morgan's laws hold true

Proof. Let $A=\{\varkappa,(\S_A(\varkappa),\imath_A(\varkappa),d_A(\varkappa))\}$, $B=\{\varkappa,(\S_B(\varkappa),\imath_B(\varkappa),d_B(\varkappa))\}$ and $C=\{\varkappa,(\S_C(\varkappa),\imath_C(\varkappa),d_C(\varkappa))\}$ be three IVTSFSs. The proof of the results is as follows:

(1) Transitivity: let us assume $A\le B$ and $B\le C$. To prove $A\le C$. As $A\le B$ so $\S_A(\varkappa)\le\S_B(\varkappa)$, $\imath_A(\varkappa)\le B_B(\varkappa)$ and $d_A(\varkappa)\ge d_B(\varkappa)$. Also, as $B\le C$ so $\S_B(\varkappa)\le\S_C(\varkappa)$, $\imath_B(\varkappa)\le\imath_C(\varkappa)$ and $d_B(\varkappa)\ge d_C(\varkappa)$.

Now, $\S_A(\varkappa)\le\S_B(\varkappa)$ and $\S_B(\varkappa)\le\S_C(\varkappa)\Longrightarrow\ \S_A(\varkappa)\le\S_C(\varkappa)$.

Also, $\imath_A(\varkappa)\le B_B(\varkappa)$ and $\imath_B(\varkappa)\le\imath_C(\varkappa)\Longrightarrow\ \imath_A(\varkappa)\le\imath_C(\varkappa)$.

Similarly, $d_A(\varkappa)\ge d_B(\varkappa)$ and $d_B(\varkappa)\ge d_C(\varkappa)\Longrightarrow d_A(\varkappa)\ge d_C(\varkappa)$.

Hence, $A\le C$.

(2) Commutativity of union: for IVTSFSs A and B,

$$\begin{aligned}A\cup B&=\{\varkappa,(\S_A(\varkappa)\vee\S_B(\varkappa),\ \imath_A(\varkappa)\wedge\imath_B(\varkappa)\text{ and }\wedge_A(\varkappa)\wedge d_B(\varkappa))\}\\&=\{\varkappa,(\S_B(\varkappa)\vee\S_A(\varkappa),\ \imath_B(\varkappa)\wedge\imath_A(\varkappa)\text{ and }\wedge_B(\varkappa)\wedge d_A(\varkappa))\}\\&=B\cup A.\end{aligned}\tag{5}$$

Commutativity of intersection is similar.

(3) Distributive law of union over intersection: to prove $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

$$\begin{aligned}A\cup(B\cap C)&=\{\varkappa,\ (\S_A(\varkappa),\ \imath_A(\varkappa),\ d_A(\varkappa))\}\cup(\{\varkappa,\ (\S_B(\varkappa)\wedge\S_C(\varkappa),\ \imath_B(\varkappa)\wedge\imath_C(\varkappa)\text{ and }d_B(\varkappa)\vee d_C(\varkappa))\})\\&=\{\varkappa,(\S_A(\varkappa)\vee(\S_B(\varkappa)\wedge t\S_C n(\varkappa)),\ \imath_A(\varkappa)\wedge(\imath_B(\varkappa)\wedge\imath_C(\varkappa)),\ d_A(\varkappa)\wedge(d_B(\varkappa)\vee d_C(\varkappa)))\}\\&=\{\varkappa,((\S_A(\varkappa)\vee t\S_B n(\varkappa))\wedge(\S_A(\varkappa)\vee\S_C(\varkappa)),\ (\imath_A(\varkappa)\vee\imath_B(\varkappa))\wedge(\imath_A(\varkappa)\vee\imath_C(\varkappa)),\ (d_A(\varkappa)\wedge d_B(\varkappa))\vee(d_A(\varkappa)\wedge d_C(\varkappa)))\}\\&=(A\cup B)\cap(A\cup C).\end{aligned}\tag{6}$$

Distributive law of intersection over union is similar to prove.

(4) De Morgan's Laws: to prove $(A\cup B)^{\mathbf{c}}=A^c\cap B^c$.

$$\begin{aligned}(A\cup B)^{\mathbf{c}}&=\{\varkappa,(\S_A(\varkappa)\vee\S_B(\varkappa),\imath_A(\varkappa)\wedge B_B(\varkappa),d_A(\varkappa)\wedge d_B(\varkappa))\}^c\\&=\{\varkappa,(d_A(\varkappa)\wedge d_B(\varkappa),\imath_A(\varkappa)\wedge\imath_B(\varkappa),\S_A(\varkappa)\vee\S_B(\varkappa))\}\\&=\{\varkappa,(d_A(\varkappa),\imath_A(\varkappa),\S_A(\varkappa))\}\cap\{\varkappa,(d_B(\varkappa),\imath_B(\varkappa),\S_B(\varkappa))\}\\&=A^c\cap B^c.\end{aligned}\tag{7}$$

Similarly, $(A \cap B)^c = A^c \cup B^c$ can be proved. □

Definition 10. For IVITSFSs $A = \{\varkappa, (\c{s}_A(\varkappa), \imath_A(\varkappa), d_A(\varkappa))\}$ and $B = \{\varkappa, (\c{s}_B(\varkappa), \imath_B(\varkappa), d_B(\varkappa))\}$, we define

$$A \oplus B = \left\{ \left\{ \varkappa, \left(\begin{array}{c} \sqrt[n]{\c{s}_A^{Ln}(\varkappa) + \c{s}_B^{Ln}(\varkappa) - \c{s}_A^{Ln}(\varkappa).\c{s}_B^{Ln}(\varkappa)}, \left[\sqrt[n]{\c{s}_A^{Un}(\varkappa) + \c{s}_B^{Un}(\varkappa) - \c{s}_A^{Un}(\varkappa).\c{s}_B^{Un}(\varkappa)} \right], \\ \left[\imath_A^L(\varkappa).\imath_B^L(\varkappa), \imath_A^U(\varkappa).\imath_B^U(\varkappa) \right], \left[d_A^L(\varkappa).d_B^L(\varkappa), d_A^U(\varkappa).d_B^U(\varkappa) \right] \end{array} \right) \right\} \right\},$$
$$A \otimes B = \left\{ \left\{ \varkappa, \left(\begin{array}{c} \left[\c{s}_A^L(\varkappa).\c{s}_B^L(\varkappa), \c{s}_A^U(\varkappa).\c{s}_B^U(\varkappa) \right], \left[\imath_A^L(\varkappa).\imath_B^L(\varkappa), \imath_A^U(\varkappa).\imath_B^U(\varkappa) \right], \\ \sqrt[n]{d_A^{Ln}(\varkappa) + d_B^{Ln}(\varkappa) - d_A^{Ln}(\varkappa).d_B^{Ln}(\varkappa)} \Big[, \sqrt[n]{d_A^{Un}(\varkappa) + d_B^{Un}(\varkappa) - d_A^{Un}(\varkappa).d_B^{Un}(\varkappa)} \Big] \end{array} \right) \right\} \right\}. \tag{8}$$

In the following, the concept of Cartesian product and relations for IVTSFSs are proposed, and their properties are studied. These operations are the generalization of operations of IVIFSs and IVPFSs.

Definition 11. The Cartesian product of two IVTSFSs A and B over two universes X_1 and X_2 is of the following form:

$$\begin{aligned} A \times_1 B &= \{(\varkappa, y), (\c{s}_A(\varkappa).\c{s}_B(y), \imath_A(\varkappa).\imath_B(y), d_A(\varkappa).d_B(y))\} \\ &= \left\{(\varkappa, y), \left(\left[\c{s}_A^L(\varkappa).t\c{s}_B^L n(y)q, h\, \c{s}_A^U x(\varkappa)7.C\c{s}_B^U; (y) \right], \left[\imath_A^L(\varkappa).\imath_B^L(y), \imath_A^U(\varkappa).\imath_B^U(y) \right], \left[d_A^L(\varkappa).d_B^L(y), d_A^U(\varkappa).d_B^U(y) \right] \right) \right\}, \\ A \times_2 B &= \{(\varkappa, y), (\c{s}_A(\varkappa) \wedge \c{s}_B(y), \imath_A(\varkappa) \wedge \imath_B(y), d_A(\varkappa) \vee d_B(y))\} \\ &= \left\{(\varkappa, y), \left(\left[\c{s}_A^L(\varkappa) \wedge t\c{s}_B^L n(y)q, h\, \c{s}_A^U x(\varkappa)7 \wedge C\c{s}_B^U; (y) \right], \left[\imath_A^L(\varkappa) \wedge \imath_B^L(y), \imath_A^U(\varkappa) \wedge \imath_B^U(y) \right], \left[d_A^L(\varkappa) \vee d_B^L(y), d_A^U(\varkappa) \vee d_B^U(y) \right] \right) \right\}. \end{aligned} \tag{9}$$

Definition 12 An IVTSFR R is an IVTSF fuzzy subset of $X \times Y$ and is of the following form:

$$R = \{(\varkappa, y), (\c{s}(\varkappa, y), \imath(\varkappa, y), d(\varkappa, y))\}, \tag{10}$$

Provided that $0 \le \c{s}^n(\varkappa, y) + \imath^n(\varkappa, y) + d^n(\varkappa, y) \le 1$ and $n \in \mathbb{Z}^+$.

Definition 13. For an IVTSFR R, R^{-1} is defined as follows:

$$R^{-1} = \left\{(y, \varkappa), \left(\c{s}^{-1}(y, \varkappa), \imath^{-1}(y, \varkappa), d^{-1}(y, \varkappa)\right)\right\}, \tag{11}$$

where
$\c{s}^{-1}(y, \varkappa) = \c{s}(\varkappa, y), \imath^{-1}(y, \varkappa) = \imath(\varkappa, y), d^{-1}(y, \varkappa) = d(\varkappa, y)$.

Proposition 1. *The following results for three IVTSFRs R, R_1, and R_2 are Boolean positive.*

(1) $(R^{-1})^{-1} = R$

(2) If $R_1 \subseteq R_2$, *then* $R_1^{-1} \subseteq R_2^{-1}$

(3) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$ *and* $(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$

(4) $R_1 \cap R_2 \subseteq R_1$ *and* $R_1 \cap R_2 \subseteq R_2$

(5) If $R_1 \subseteq R$ *and* $R_2 \subseteq R$, *then* $R_1 \cup R_2 \subseteq R$

(6) If $R \subseteq R_1$ *and* $R \subseteq R_2$, *then* $R \subseteq R_1 \cap R_2$

Definition 14. For two IVTSFRs R_1 on $X \times Y$ and R_2 on $Y \times Z$, their composition is denoted by $R_2 \circ R_1$ and defined as follows:

$$R_2 \circ R_1 = \{(\varkappa, z), (\c{s}(\varkappa, z), \imath(\varkappa, z), d(\varkappa, z))\}, \tag{12}$$

where $\c{s}(\varkappa, z) = \vee\{(\c{s}(\varkappa, y) \wedge \c{s}\ (y, z))\} = \vee_y \{([\c{s}^L(\varkappa, y) \wedge \c{s}^L(y, z), \c{s}^U(\varkappa, y) \wedge \c{s}^{Uy}(y, z)])\}$,

$$\begin{aligned} \imath(\varkappa, z) &= \wedge_y \{(\imath(\varkappa, y) \wedge \imath(y, z))\} \\ &= \wedge_y \left\{\left(\left[\imath^L(\varkappa, y) \wedge \imath^L(y, z), \imath^U(\varkappa, y) x \imath^U(y, z)\right]\right)\right\}, \\ d(\varkappa, z) &= \wedge_y \{(d(\varkappa, y) \vee d(y, z))\} \\ &= \wedge_y \left\{\left(\left[d^L(\varkappa, y) \vee d^L(y, z), d^U(\varkappa, y) \vee d^U(y, z)\right]\right)\right\}. \end{aligned} \tag{13}$$

Definition 15. The score function for an IVTSFN $A = ([\mathscr{Z}^L, \mathscr{Z}^U], [\check{r}^L, \dot{\imath}^U], [\eta^L, \eta^U])$ is defined as follows:

$$\mathrm{SC}(A) = \frac{(\mathscr{Z}^L)^n \left(1 - (\check{r}^L)^n - (\eta^L)^n\right) + (\mathscr{Z}^U)^n \left(\left(1 - (\dot{\imath}^U)^n - (\eta^U)^n\right)\right)}{3}, \mathrm{SC}(A) \in [0, 1] \tag{14}$$

Remark 1. Replacing $\imath^L = \imath^U = 0$ and $n = 1$ reduces the defined function for score calculation.

4. Importance of Structures: Interval-Valued Fuzzy

The worthy concept of modeling a MG of a FS using an interval was the inception of Gorzalczany [2], whereas the concepts of IFS, PyFS, Q-ROPFS, and PFS are established in [5, 6, 15, 50]. The IVFS's concept has shown great significance, especially in those situations where an occurrence is difficult to be attributed by a crisp number and therefore, the significance of interval valued frameworks is elaborated in more detail.

The main decision in the following instance is regarding the selection of the best candidate among five candidates ($\dot{g}_i$, $i = 1, 2, 3, 4$), with six attributes ($\dot{g}_j$, $j = 1, 2, 3, 4$) and weight vector $w = (0.22, 0.34, 0.27, 0.17)^T$. In this case, Table 1 illustrates the evaluation data.

The proposed aggregation tool by Xu and Cai [29] for the data provided in Table 1 is as follows:

$$\dot{g}_i = \text{IVIFWA}(\dot{g}_1, \dot{g}_2, \dot{g}_3, \ldots, \dot{g}_m) = \left(\left[1 - \prod_{j=1}^{m}\left(1 - \left(\mathcal{Z}_j^l\right)\right)^{w_j}, 1 - \prod_{j=1}^{m}\left(1 - \left(\mathcal{Z}_j^u\right)\right)^{w_j}\right], \left[\prod_{j=1}^{m}\left(\eta_j^l\right)^{w_j}, \prod_{j=1}^{m}\left(\eta_j^u\right)^{w_j}\right]\right). \quad (15)$$

The aggregated results are as follows:

$$\begin{aligned}
\dot{g}_1 &= \text{IVIFWA}(\dot{g}_{11}, \dot{g}_{12}, \dot{g}_{13}, \dot{g}_{14}) = ([0.6235, 0.7569], [0.0000, 0.3386]),\\
\dot{g}_2 &= \text{IVIFWA}(\dot{g}_{21}, \dot{g}_{22}, \dot{g}_{23}, \dot{g}_{24}) = ([0.6777, 0.8180], [0.0000, 0.2800]),\\
\dot{g}_3 &= \text{IVIFWA}(\dot{g}_{31}, \dot{g}_{32}, \dot{g}_{33}, \dot{g}_{34}) = ([0.5730, 0.7745], [0.1713, 0.3454]),\\
\dot{g}_4 &= \text{IVIFWA}(\dot{g}_{41}, \dot{g}_{42}, \dot{g}_{43}, \dot{g}_{44}) = ([0.6493, 0.8352], [0.1165, 0.2510]).
\end{aligned} \quad (16)$$

The score of IVFN $I = ([\eta^l, \eta^u], [\eta^l, \eta^u])$ is defined as follows [31]:

$$\begin{aligned}
\text{SC0}(I) &= \frac{1}{2}\left(\mathcal{Z}^l - \eta^l + \mathcal{Z}^u - \eta^u\right),\\
\text{SC}(\dot{g}_1) &= 0.5209, \text{SC}(\dot{g}_2) = 0.6079, \text{SC}(\dot{g}_3)\\
&= 0.4154, \text{SC}(\dot{g}_4) = 0.5585.
\end{aligned} \quad (17)$$

Clearly, $\text{SC0}(\dot{g}_3) > \text{SC0}(\dot{g}_4) > \text{SC0}(\dot{g}_1) > \text{SC} - (\dot{g}_2)$

Hence, according to the MADM method, $\dot{g}_2$ is considered to be the most appropriate selection candidate. The assignment of a crisp value to membership and NMG causes the information loss in Table 1 and can be aggregated using the aggregation operators Xu [22]. The decision matrix is illustrated in Table 2.

The weighted averaging aggregation operator of IFSs, proposed by Xu [22], is given by the following:

$$\begin{aligned}
\dot{g}_1 &= \text{IFWA}(\dot{g}_{11}, \dot{g}_{12}, \dot{g}_{13}, \ldots \dot{g}_m) = \left(1 - \prod_{j=1}^{m}\left(1 - \left(\mathcal{Z}_j^l\right)\right)^{w_j}, \prod_{j=1}^{m}\left(d_j^l\right)^{w_j}\right),\\
\dot{g}_1 &= \text{IFWA}(\dot{g}_{11}, \dot{g}_{12}, \dot{g}_{13}, \dot{g}_{14}) = (0.7569, 0.3386),\\
\dot{g}_2 &= \text{IFWA}(\dot{g}_{21}, \dot{g}_{22}, \dot{g}_{23}, \dot{g}_{24}) = (0.8180, 0.2800),\\
\dot{g}_3 &= \text{IFWA}(\dot{g}_{31}, \dot{g}_{32}, \dot{g}_{33}, \dot{g}_{34}) = (0.7745, 0.3454),\\
\dot{g}_4 &= \text{IFWA}(\dot{g}_{41}, \dot{g}_{42}, \dot{g}_{43}, \dot{g}_{44}) = (0.7489, 0.1165).
\end{aligned} \quad (18)$$

For ranking IFN I, the score function developed in [22] is given by $\text{SC0}(I) = \mathcal{Z} - \eta$.

Using this score functions, we have the following:

$$\begin{aligned}
\text{SC0}(\dot{g}_1) &= 0.0361, \text{SC0}(\dot{g}_2) = 0.2229, \text{SC0}(\dot{g}_3)\\
&= 0.1279, \text{SC0}(\dot{g}_4) = 0.3007.
\end{aligned} \quad (19)$$

Clearly, $\text{SC0}(\dot{g}_4) > \text{SC0}(\dot{g}_2) > \text{SC0}(\dot{g}_3) > \text{SC}(\dot{g}_1)$

Thus, according to the MADM approach, C_4 is the appropriate candidate with IVIFWA operators. The analysis of the results revealed the significance of the aggregation of IVIFS in contrast to the one obtained using IFS. The concept of TSFS [49] is a generalization of Q-ROPFS, SFS, PyFS, PFS, IFS, as well as FS, and therefore, the interval-valued structure for TSFSs with its diverse structure would have an outstanding impact. The concept of IVTSFS [6, 7, 49] gives further convincing strength to the notion of TSFSs due to its usefulness and flexibility towards the assignment of MGs of TSFS in terms of intervals.

Table 1: Decision matrix: evaluation data.

	$\dot{g}_1$	$\dot{g}_2$	$\dot{g}_3$	$\dot{g}_4$
$\dot{g}_1$	([0.2, 0.3], [0.0, 0.5])	([0.5, 0.6], [0.1, 0.3])	([0.4, 0.5], [0.2, 0.4])	([0.7, 0.8], [0.1, 0.2])
$\dot{g}_2$	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.6], [0.0, 0.3])	([0.6, 0.7], [0.2, 0.3])	([0.6, 0.7], [0.1, 0.2])
$\dot{g}_3$	([0.4, 0.5], [0.3, 0.4])	([0.3, 0.6], [0.1, 0.3])	([0.5, 0.6], [0.3, 0.4])	([0.6, 0.7], [0.1, 0.3])
$\dot{g}_4$	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.7], [0.1, 0.3])	([0.7, 0.8], [0.1, 0.2])	([0.3, 0.4], [0.1, 0.2])

Table 2: Decision matrix.

	$\dot{g}_2$	$\dot{g}_2$	$\dot{g}_2$	$\dot{g}_2$
$\dot{g}_1$	(0.3, 0.5)	(0.6, 0.3)	(0.5, 0.4)	(0.8, 0.2)
$\dot{g}_2$	(0.7, 0.3)	(0.6, 0.3)	(0.7, 0.3)	(0.7, 0.2)
$\dot{g}_3$	(05, 0.4)	(0.6, 0.3)	(0.6, 0.4)	(0.7, 0.3)
$\dot{g}_4$	(0.6, 0.2)	(0.5, 0.1)	(0.7, 0.1)	(0.3, 0.1)

5. Interval-Valued T-Spherical Fuzzy Graphs

The IVTSFS is a generalization of FS, IFSs, PFSs, IVIFSs, and IVPFSs, and therefore, it is of great importance. The notion of graph for IVTSFS, i.e., IVTSFG is proposed as a generalization of FG, IFG, PFG, IVIFG, and IVPFG along with fundamental terms and definitions with applications in Section 6.

Definition 16. An IVTSFG is $= (V, E)$, V is node set and E edges' collection and

(1) For all $v \in V$ is attributed with three functions ş, $\imath$ and d degree of membership, abstinence, and nonmembership of $v \in V$. Basically, $\text{ş} = [\text{ş}^L, \text{ş}^L]$, $\imath = [\imath^L, \imath^U]$ and $d = [d^L, d^L]$ are subintervals of the unit interval $[0, 1]$ with a condition that $0 \leq (\text{ş}^U)^n + (\imath^U)^n + (d^U)^n \leq 1$ for $n \in \mathbb{Z}^+$. The $R = [R^U, R^L]$ denotes the refusal degree of $v \in V$ and $R^U = \sqrt[n]{1 - (\text{ş}^L)^n - (\imath^L)^n - (d^L)^n}$ and $R^L = \sqrt[n]{1 - (\text{ş}^U)^n - (\imath^U)^n - (d^U)^n}$.

(2) For all $e \in E$ is attributed with three functions $\widehat{S}$, $\hat{I}$ and Đ degree of membership, abstinence, and nonmembership of $e = (\widehat{S}, \hat{I}, \text{Đ}) \in V \times V$. Basically $\widehat{S} = [\widehat{S}^L, \widehat{S}^L]$, $\hat{I} = [\hat{I}^L, \hat{I}^U]$ and $\text{Đ} = [\text{Đ}^L, \text{Đ}^L]$ are as follows:
$\widehat{S}^L(v_i, v_j) \leq \min(\text{ş}^L(v_i), \text{ş}^L(v_j))$ and $\widehat{S}^U(v_i, v_j) \leq \min(\text{ş}^U(v_i), \text{ş}^U(v_j))$ s.t $\widehat{S}^U \geq \min(\text{ş}^L(v_i), \text{ş}^L(v_j))$.
$\hat{I}^L(v_i, v_j) \leq \min(\imath^L(v_i), \imath^L(v_j))$ and $\hat{I}^U(v_i, v_j) \leq \max(\imath^U(v_i), \imath^U(v_j))$ s.t $\hat{I}^U \geq \max(\imath^L(v_i), \imath^L(v_j))$.
$\text{Đ}^L(v_i, v_j) \leq \max(d^L(v_i), d^L(v_j))$ and $\text{Đ}^U(v_i, v_j) \leq \max(d^U(v_i), d^U(v_j))$ s.t $\text{Đ}^U \geq \max(d^L(v_i), d^L(v_j))$.
With a condition $0 \leq (\widehat{S}^U)^n + (\hat{I}^U(\varkappa))^n + (\text{Đ}^U(\varkappa))^n \leq 1$ for $n \in \mathbb{Z}^+$, the term $\check{R} = [\check{R}^U, \check{R}^L]$ denotes the refusal degree of $e \in E$ and $\check{R}^U = \sqrt[n]{1 - (\widehat{S}^L)^n - (\hat{I}^L)^n - (\text{Đ}^L)^n}$ and $\check{R}^L = \sqrt[n]{1 - (\widehat{S}^U)^n - (\hat{I}^U)^n - (\text{Đ}^U)^n}$.

Theorem 3. *IVTSFG is a generalization of IVPFG, IVIFG, and IVFG.*

(1) If we take $n = 1$, the IVTSFG diminishes to IVPFG

(2) If we take $n = 1$ and $\imath^L = \imath^U = 0$, the IVTSFG diminishes to IVIFG

(3) If we take $n = 1$, $\imath^L = \imath^U = 0$ and $d^L = d^U = 0$, the IVTSFG diminishes to IVFG

The outcome reveals the significance of the novel concept due to its generalization.

Example 3. The following Figure 6 is an IVTSFG with a set of nodes v_i $(i = 1, 2, 3, \ldots, 6)$ and a set of edges e_i i $(i = 1, 2, 3, 4$ and $5))$.

Definition 17. An IVTSFG $G' = (V', E')$ is IVTSFSG of graph $G = (V, E)$ iff $V' \subseteq V$ and $E' \subseteq E$

Example 4. The IVTSFSG of IVTSFG provided in Figure 6 is depicted in Figure 7

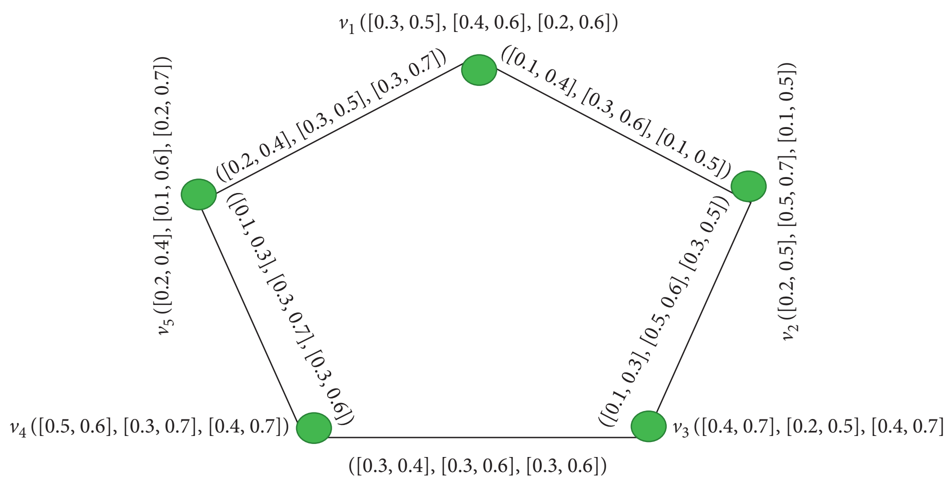

Figure 6: IV T-spherical fuzzy graph.

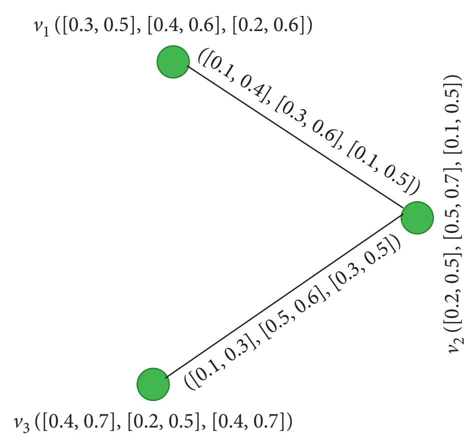

Figure 7: Interval-valued T-spherical fuzzy subgraph.

Definition 18. The complement, i.e., IVTSFG $G = (V, E)$ is represented by $G^c = (V^c, E^c)$ where $V^c = V$ and the MGs of E are defined by the following:

$$\begin{aligned}
\left(\widehat{S}^{L}\right)^{c}\left(v_i, v_j\right) &= \min\left(ş^{L}(v_i), ş^{L}\left(v_j\right)\right) - \widehat{S}^{L}\left(\left(v_i, v_j\right)\right),\\
\left(\widehat{S}^{U}\right)^{c}\left(v_i, v_j\right) &= \min\left(ş^{U}(v_i), ş^{U}\left(v_j\right)\right) - \widehat{S}^{U}\left(v_i, v_j\right) + \min\left(ş^{L}(v_i), ş^{L}\left(v_j\right)\right),\\
\left(\hat{\mathrm{I}}^{\mathrm{L}}\right)^{c}\left(v_i, v_j\right) &= \min\left(\imath^{\mathrm{L}}(v_i), \imath^{\mathrm{L}}\left(v_j\right)\right) - \hat{\mathrm{I}}^{\mathrm{L}}\left(\left(v_i, v_j\right)\right),\\
\left(\hat{\mathrm{I}}^{\mathrm{U}}\right)^{c}\left(v_i, v_j\right) &= \min\left(\imath^{\mathrm{U}}(v_i), \imath^{\mathrm{U}}\left(v_j\right)\right) - \hat{\mathrm{I}}^{\mathrm{U}}\left(v_i, v_j\right) + \max\left(\imath^{\mathrm{L}}(v_i), \imath^{\mathrm{L}}\left(v_j\right)\right),\\
\left(Đ^{L}\right)^{c}\left(v_i, v_j\right) &= \max\left(d^{L}(v_i), d^{L}\left(v_j\right)\right) - Đ^{L}\left(v_i, v_j\right),\\
\left(Đ^{U}\right)^{c}\left(v_i, v_j\right) &= \max\left(d^{U}(v_i), d^{U}\left(v_j\right)\right) - Đ^{U}\left(v_i, v_j\right) + \max\left(d^{L}(v_i), d^{L}\left(v_j\right)\right).
\end{aligned} \tag{20}$$

Theorem 4. *For IVTSFG $G = (V, E)$, $(G^c)^c = G$.*

Proof. Let $G = (V, E)$ be an IVTSFG. The proof is as follows:

$$\begin{aligned}
\left(\widehat{S}^L\right)^c(v_i, v_j) &= \min\left(ş^L(v_i), ş^L(v_j)\right) - \widehat{S}^L\left((v_i, v_j)\right),\\
\left(\left(\widehat{S}^L\right)^c\right)^c(v_i, v_j) &= \min\left(ş^L(v_i), ş^L(v_j)\right) - \left(\min\left(ş^L(v_i), ş^L(v_j)\right) - \widehat{S}^L\left((v_i, v_j)\right)\right) = \widehat{S}^L\left((v_i, v_j)\right),\\
\left(\widehat{S}^U\right)^c(v_i, v_j) &= \min\left(ş^U(v_i), ş^U(v_j)\right) - \widehat{S}^U(v_i, v_j) + \min\left(ş^L(v_i), ş^L(v_j)\right),\\
\left(\left(\widehat{S}^U\right)^c\right)^c(v_i, v_j) &= \min\left(ş^U(v_i), ş^U(v_j)\right) - \left(\min\left(ş^U(v_i), ş^U(v_j)\right) - \widehat{S}^U(v_i, v_j) + \min\left(ş^L(v_i), ş^L(v_j)\right)\right) + \min\left(ş^L(v_i), ş^L(v_j)\right).
\end{aligned} \tag{21}$$

$((\widehat{S}^U)^c)^c(v_i, v_j) = \widehat{S}^U(v_i, v_j)$. Similarly, we can prove that

$$\begin{aligned}
\left(\left(\hat{I}^L\right)^c\right)^c(v_i, v_j) &= \hat{I}^L(v_i, v_j),\\
\left(\left(\hat{I}^U\right)^c\right)^c(v_i, v_j) &= \hat{I}^U(v_i, v_j),\\
\left(\left(Đ^L\right)^c\right)^c(v_i, v_j) &= Đ^L(v_i, v_j),\\
\left(\left(Đ^U\right)^c\right)^c(v_i, v_j) &= Đ^U(v_i, v_j), \quad \text{Hence } (G^c)^c = G.
\end{aligned} \tag{22}$$

□

Example 5. Figure 8 represents the IVTSFGs graph, whereas Figure 9 represents its complement.

Definition 19. An IVTSFG is known as follows:

(1) Semi S strong: if $\widehat{S}^L(v_i, v_j) = \min(ş^L(v_i), ş^L(v_j))$ and $\widehat{S}^U(v_i, v_j) = \min(ş^U(v_i), ş^U(v_j))$
(2) Semi I strong: if $\hat{I}^L(v_i, v_j) = \min(ı^L(v_i), ı^L(v_j))$ and $\hat{I}^U(v_i, v_j) = \min(ı^U(v_i), ı^U(v_j))$
(3) Semi D strong: if $Đ^L(v_i, v_j) = \max(d^L(v_i), d^L(v_j))$ and $Đ^U(v_i, v_j) = \max(d^U(v_i), d^U(v_j))$
(4) Strong: if (1), (2), and (3) hold true.

Example 6. Figure 10 represents IVTSFG.

Definition 20. In an IVTSFG, a set of nonidentical nodes $v_i\,(i = 1, 2, , 3, \ldots, m)$ is regarded as a path provided an edge exists between every two vertices v_i and v_j for $i, j = 1, 2, , 3, \ldots, m$. The nonidentical nodes $v_i\,(i = 1, 2, , 3, \ldots, m)$ are equivalent to a path provided at least one of the following conditions holds true.

(1) $\widehat{S}(v_i, v_j)$ is a nonzero subinterval of $[0, 1]$
(2) $\hat{I}(v_i, v_j)$ is a nonzero subinterval of $[0, 1]$
(3) $Đ(v_i, v_j)$ is a nonzero subinterval of $[0, 1]$

Consequences of Definition 20:

(1) The length of a path is m−1 if it has m vertices
(2) If the first and last vertices of a path coincide, then it will be a cycle
(3) If two vertices are joined by a path, then they will be regarded as connected

Example 7. In Figure 11,

(1) $v_5v_1v_4v_2v_3$ is a path
(2) $v_1v_4v_2$ is a cycle

6. Applications

Several applications of IVTSFSs and IVTSFGs in anomaly detection, medical diagnosis, and SP problem are discussed in this section. The comparative assessment of the results is also presented.

6.1. Anomaly Detection. The Anomaly/Malware based threat has become an irresistible concern for businesses and organizations. Malware is an abbreviated form of "malicious software." It is a file or a set of instructions intended to bring controlled or catastrophic damage to organizations, facilities, processes, cloud infrastructure, industrial processes, and digital systems. In 2018, the number of users attacked with banking Trojans was 889,452, an increase of 15.9% in comparison with 767,072 in 2017, whereas in 2019, more than 100 million different hosts were attacked in H1 2019 as reported by Kaspersky Labs. Moreover, as per the statistics collected during research conducted by Juniper revealed that the cost of data breaches would rise from \$3 trillion each year to over \$5 trillion in 2024. Therefore, anomaly detection is one of the most critical elements for cyber security. As IVTSFS is a generalization of TSFSs and IVIFSs, therefore we used the approach of IVTSFSs using aggregation operators to handle anomaly detection problem that is proved to be of great significance. An anomaly is considered to be a file with malicious content or it can be Portable Executables (PE) file. In order to make a decision regarding anomaly detection problems, i.e., information is obtained about a given PE file and associated attributes/features are represented in an appropriate format for subsequent analysis. After the representation, aggregation operators are applied by the problem solvers. Subsequently, the score value is computed using the score function for IVTSFSs to make a decision regarding the classification of the anomaly, i.e., PE

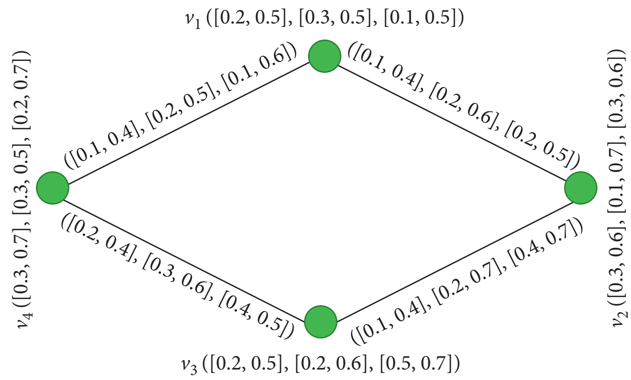

Figure 8: Interval-valued T-spherical fuzzy graph.

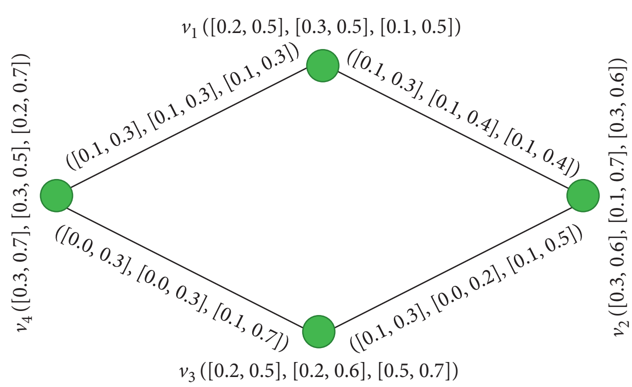

Figure 9: Complement of Figure 8.

sample either as malicious or benign. The algorithm for malware detection is shown in Algorithm 1, and the algorithm for medical diagnosis is shown in Algorithm 2.

Consider a sample software file S being investigated by anomaly analyst under k attributes as A_k. The problem solvers investigate the a sample file under k attributes and represent the findings in the form of IVTSFNs. Further, $w = (w_1, w_2, w_3, \ldots, w_k)^T$ corresponds to the weight vector of attributes. The classification process is given as follows:

The proposed algorithm is illustrated in Figure 12.

Example 8. Consider a multinational cyber security consultancy firm who has to make a decision about the classification of a given zero-day anomaly. A zero-day malware/anomaly is an anomaly/malware one that is either unknown or not yet addressed by anyone. There is a team of domain experts in the research lab of that consultancy firm to investigate the problem and make a decision regarding the classification, i.e., malicious or benign (not malicious) of a given sample.

Initially, assume there are four samples to be examined. S_1: Windows-based PE Sample # 1, S_2: Windows-based PE Sample # 2, S_3: Windows-based PE Sample # 3, and S_4: Windows-based PE Sample # 4. In order to correctly analyze the samples, their attributes/features have to be extracted. So for this purpose, there are four features to be extracted and analyzed: A_1: entropy of the PE sample, A_2: resource size of the PE sample, A_3: virtual Size of the PE sample, and A_4: section mean entropy of the PE sample. All the four attributes are most significant ones for anomaly detection. Let $w = (0.23, 0.35, 0.26, 0.16)^T$ be the weight vector and represent their opinions in IVTSFNs format. The stepwise demonstration of the process is as follows:

Step 1: in the first step, the analyst examines the given samples S_i and extracts the four attributes/features including entropy, resource size, virtual size, and section mean entropy. After the extraction of the aforementioned four attributes (A_1, A_2, A_3, and A_4) from given PE samples (S_1, S_2, S_3, and S_4), a decision matrix $D_{4\times 4}$ is constituted. The representation of attributes (A_1, A_2, A_3, and A_4) of PE samples (S_1, S_2, S_3, and S_4) in matrix $D_{4\times 4}$ is given as in Table 3:

The assessment analysis of individual score against the threshold shows that Score of S_3 is greater than given criteria, i.e., 0.5; therefore, only S_3 among all samples is declared as malicious, i.e., anomaly. Moreover, as the score of S_1, S_2, and S_4 is below the defined threshold criteria, i.e., 0.5; therefore, they will be classified as benign in nature. Hence, it is shown that how can a critical and complex problem; i.e., Anomaly Detection can be solved using the proposed intelligent and robust decision making approach.

The classification decision of the given sample (s) either as malicious or benign is carried out by using the IVTSFWA operators. Only for $q = 5$ values are IVTSFNs.

Step 2: the IVTSFWA operators to the matrix gives the following output:

$$S_1 = \text{IVTSFWA}(S_{11}, S_{12}, S_{13}, S_{14}) = ([0.4698, 0.8600], [0.2700, 0.4000], [0.2833, 0.3866]),$$
$$S_2 = \text{IVTSFWA}(S_{21}, S_{22}, S_{23}, S_{24}) = ([0.3836, 0.7655], [0.2703, 0.5060], [0.4257, 0.5579]), \quad (23)$$
$$S_3 = \text{IVTSFWA}(S_{31}, S_{32}, S_{33}, S_{34}) = ([0.7470, 0.9000], [0.0000, 0.2382], [0.2229, 0.3336]),$$

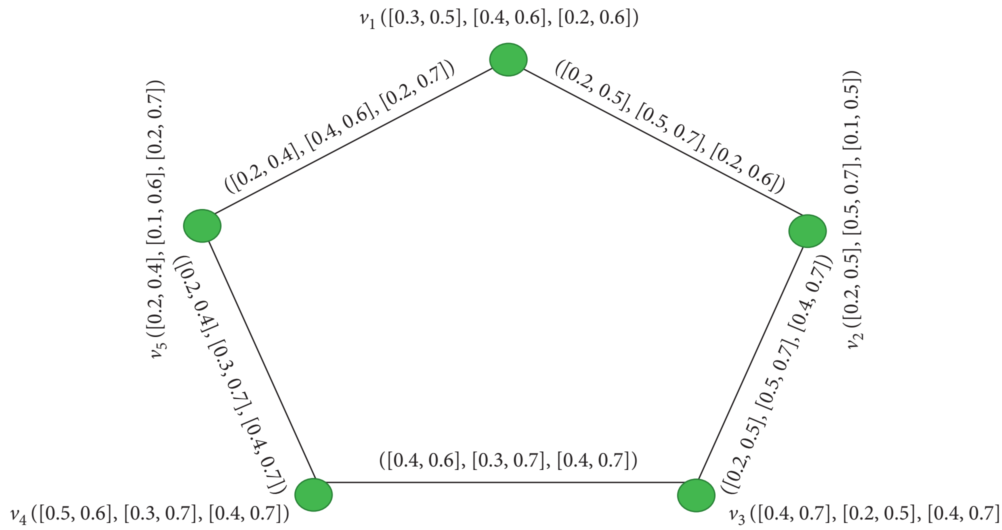

Figure 10: Strong interval-valued T-spherical fuzzy graph.

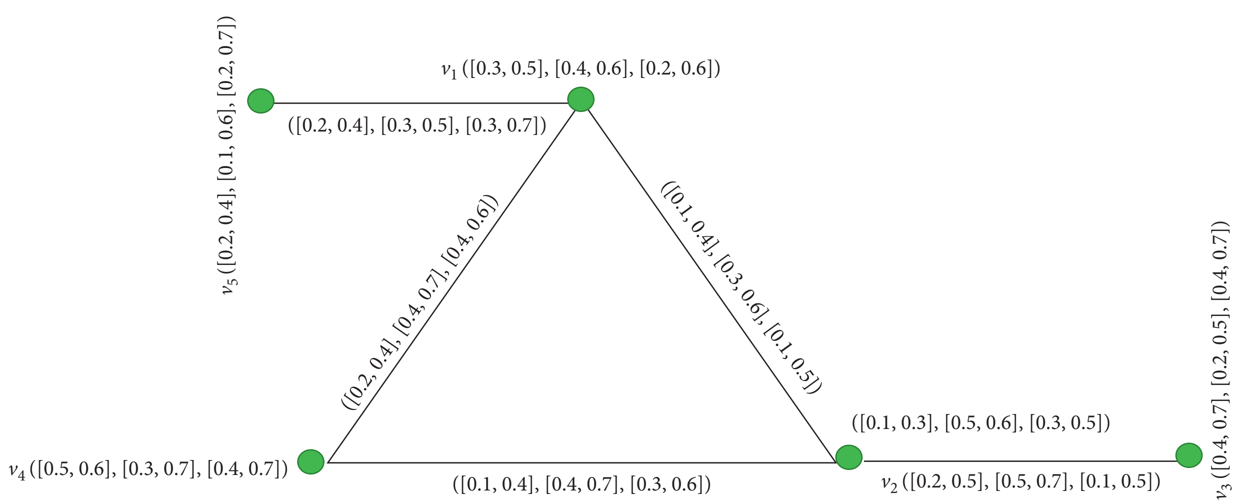

Figure 11: IV T-spherical fuzzy graph.

(1) Analyst will perform the extraction of features/attributes (A) from the given PE sample (S) and constitute a decision Matrix.
(2) Put on the following aggregation tools to the attribute matrix obtained in Step 1.
$\text{IVTSFWA}(I_1, I_2, I_3 \ldots I_m) = ([\sqrt[q]{1-\prod_{j=1}^{m}(1-(\c{s}_j^{l})^{q})^{w_j}}, \sqrt[q]{1-\prod_{j=1}^{m}(1-(\c{s}_j^{u})^{q})^{w_j}}], [\prod_{j=1}^{m}(i_j^{l})^{w_j}, \prod_{j=1}^{m}(i_j^{u})^{w_j}], [\prod_{j=1}^{m}(d_j^{l})^{w_j}, \prod_{j=1}^{m}(d_j^{u})^{w_j}]),$
where I1, I2, I3 . . .Im are interval-valued T-spherical fuzzy numbers.
(3) Calculate the rank/score of IVTSFNs acquired in the previous step.
(4) Analyze the score of the given PE sample, i.e., SC (S) takes the classification decision based upon the following threshold based criteria:
SC (Sample) > 0.5 then Classification = M i.e. Malicious else Classification = B i.e. Benign

Algorithm 1: Algorithm for malware detection.

(1) Formation of relation R_1: $\alpha \longrightarrow \beta$
(2) Formation of relation R_2: $\beta \longrightarrow \gamma$
(3) Computation of R_3: $\alpha \longrightarrow \gamma$ as a composition of R_1 and R_2
(4) Investigation of diagnosis for patients using the score function

Algorithm 2: Algorithm for medical diagnosis.

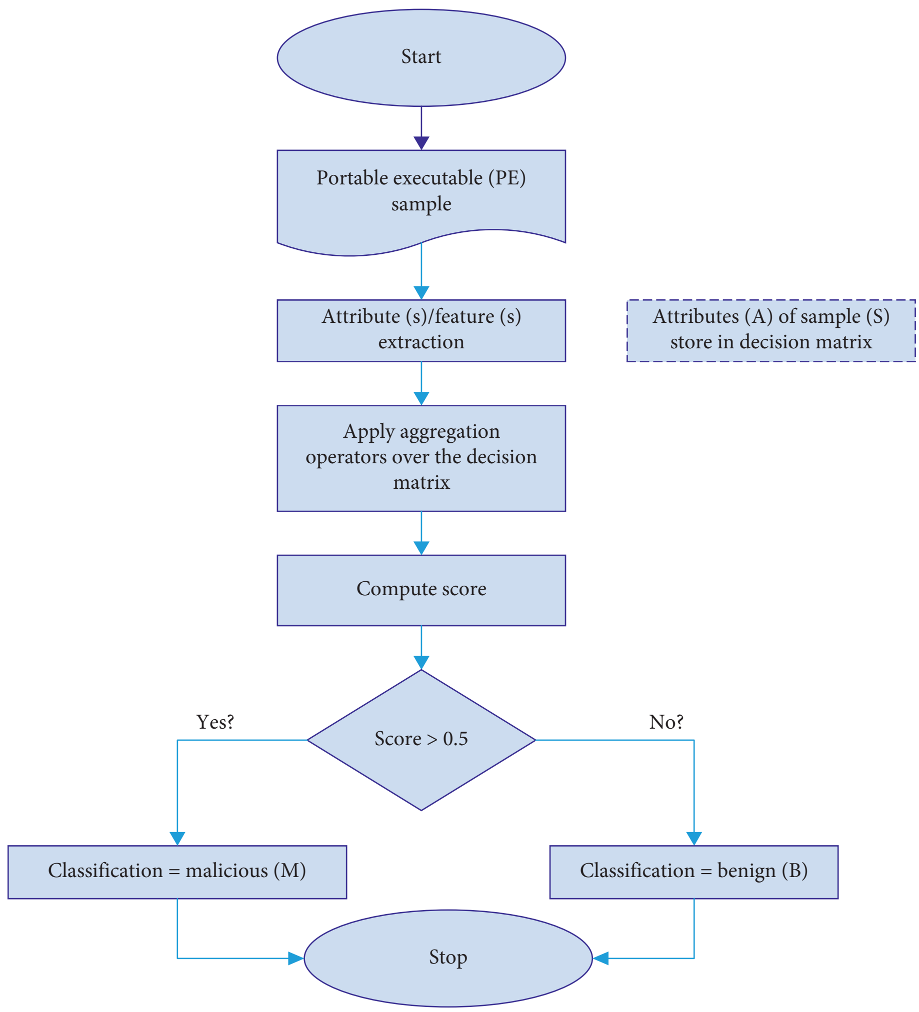

Figure 12: Flowchart describing the proposed anomaly detection process.

Step 3: score values are computed as follows:

$$\begin{aligned}
&\mathrm{SC}(S_1) = 0.2855, SC(S_2) = 0.1551, \mathrm{SC}(S_3) = 0.6429, \text{and}, \mathrm{SC}(S_4) = 0.1269,\\
&\mathrm{SC}(S_1) < 0.5;\ \mathrm{SC}(S_2) < 0.5,\\
&\quad S_4 = \mathrm{IVTSFWA}(S_{41}, S_{42}, S_{43}, S_{44}) = ([0.2090, 0.7814], [0.3818, 0.5595], [0.3426, 0.4288]),\\
&\mathrm{SC}(S_3) > 0.5.
\end{aligned} \tag{24}$$

Table 3: Decision matrix: anomaly detection.

	A_1	A_2	A_3	A_5
S_1	([0.3, 0.9], [0.3, 0.4], [0.2, 0.3])	([0.4, 0.8], [0.3, 0.4], [0.4, 0.5])	([0.5, 0.9], [0.2, 0.4], [0.3, 0.4])	([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])
S_2	([0.2, 0.5], [0.4, 0.6], [0.7, 0.8])	([0.4, 0.8], [0.3, 0.6], [0.5, 0.8])	([0.3, 0.8], [0.2, 0.4], [0.2, 0.2])	([0.8, 0.9], [0.0, 0.1], [0.5, 0.6])
S_3	([0.8, 0.9], [0.0, 0.2], [0.2, 0.3])	([0.7, 0.9], [0.1, 0.2], [0.3, 0.4])	([0.7, 0.9], [0.5, 0.6], [0.1, 0.2])	([0.8, 0.9], [0.0, 0.1], [0.5, 0.6])
S_4	([0.1, 0.2], [0.6, 0.7], [0.3, 0.4])	([0.2, 0.8], [0.3, 0.6], [0.3, 0.3])	([0.1, 0.9], [0.3, 0.4], [0.5, 0.6])	([0.3, 0.4], [0.5, 0.6], [0.3, 0.6])

6.1.1. Comparative Analysis. The resolution of comparative assessment reiterates that none of the existing structures including FSs, IVFSs, IFSs, IVIFSs, PyFSs, IVPyFSs, q-OFS, IVq-OFS, PFSs, and IVPFSs processed the data except the proposed approach and its effectiveness is demonstrated by solving the similar problem using the existing IVq-OFNs method in Table 4.

In this section, previously discussed four similar Windows based PE samples are used. The decision is required regarding their anomaly classification, i.e., either as malicious or benign effectively addressed using the aggregation operators of IVq-OFNs by taking $\mathbf{q} = 2$ and $\mathbf{i^l} = \mathbf{i^u} = \mathbf{i} = 0$. The IVTSFWA operator for $\mathbf{q} = 2$ and $\mathbf{i^l} = \mathbf{i^u} = \mathbf{i} = 0$ becomes as follows:

$$= \left(\left[\sqrt[q]{1-\prod_{j=1}^{m}\left(1-\left(\text{ş}_j^l\right)^q\right)^{w_j}}, \sqrt[q]{1-\prod_{j=1}^{m}\left(1-\left(\text{ş}_j^u\right)^q\right)^{w_j}}\right], \left[\prod_{j=1}^{m}\left(d_j^l\right)^{w_j}, \prod_{j=1}^{m}\left(d_j^u\right)^{w_j}\right]\right) \quad (25)$$

The aggregation results are as follows:

$$\begin{aligned} S_1 &= \text{IVTSFWA}\left(S_{11}, S_{12}, S_{13}, S_{14}\right) = \left([0.4698, 0.8600], [0.2833, 0.3866]\right),\\ S_2 &= \text{IVTSFWA}\left(S_{21}, S_{22}, S_{23}, S_{24}\right) = \left([0.3836, 0.7655], [0.4257, 0.5579]\right),\\ S_3 &= \text{IVTSFWA}\left(S_{31}, S_{32}, S_{33}, S_{34}\right) = \left([0.7470, 0.9000], [0.2229, 0.3336]\right),\\ S_4 &= \text{IVTSFWA}\left(S_{41}, S_{42}, S_{43}, S_{44}\right) = \left([0.2090, 0.7918], [0.3426, 0.4288]\right). \end{aligned} \quad (26)$$

Using the score function, we get the following:

$$\begin{aligned} SC\left(S_1\right) &= 0.5175, SC\left(S_2\right) = 0.3223, SC\left(S_3\right)\\ &= 0.8902, \text{ and } SC\left(S_4\right) = 0.2944. \end{aligned} \quad (27)$$

The analysis of each given sample is summarized as follows:

$$\begin{aligned} \text{SC}\left(S_1\right) > 0.5;\ \text{SC}\left(S_3\right) > 0.5,\\ \text{SC}\left(S_2\right) < 0.5, \text{SC}\left(S_4\right) < 0.5. \end{aligned} \quad (28)$$

The assessment analysis of individual scores shows that Score of S_1 and S_3 is greater than given criteria, i.e., 0.5; therefore only S_1 and S_3 samples are declared as Malicious file, i.e., anomaly. Moreover, as the Score of S_2 and S_4 is below the defined threshold, i.e., 0.5; therefore, they will be classified as benign.

Comparing the respective score of all samples in both sections reveals an instance of wrong judgement about sample S_1. The classification of anomaly sample S_1 by using the IVq-OFNs method is Malicious which was previously classified as benign. The sample S_1 is actually benign. This occurrence of wrong judgement about sample S_1 is also known as False Positive; i.e., a benign PE file is classified as malicious. Therefore, the proposed claim using IVTSFWA operators is a worthy concept.

6.2. Medical Diagnosis. In this subsection, the process of medical diagnosis is carried out in the context of IVTSFSs. The medical diagnosis process based on TSFSs is established in [6] and proved to be of great importance as IFSs and PFSs failed to process such type of information. As IVTSFS is a generalization of TSFSs and IVIFSs so we used the approach of IVTSFRs to handle medical diagnosis problem.

In a medical diagnosis problem, some information is obtained about patients and symptoms and symptoms and diagnosis in terms of IVTSFRs showing the relation between them. Then, with the help of the composition of IVTSFRs, a relation between patient and diagnosis is obtained which is further analyzed using the score function for IVTSFSs. The proposed medical diagnosis problem is demonstrated through the following algorithm where α, β and γ represents a set of patients, symptoms, and diagnosis, respectively.

The proposed algorithm is depicted in Figure 13.

Example 9. The example that we are discussing here is adapted from [6], and instead of using TSFNs, we used

Table 4: Comparative analysis: decision matrix in the form of IVq-OFNs.

$$D4 \times 4 = \begin{pmatrix} & A_1 & A_2 & A_3 & A_4 \\ S_1 & \begin{pmatrix}[0.3, 0.9],\\ [0.2, 0.3]\end{pmatrix} & \begin{pmatrix}[0.4, 0.8],\\ [0.4, 0.5]\end{pmatrix} & \begin{pmatrix}[0.5, 0.9],\\ [0.3, 0.4]\end{pmatrix} & \begin{pmatrix}[0.6, 0.8],\\ [0.2, 0.3]\end{pmatrix} \\ S_2 & \begin{pmatrix}[0.2, 0.5],\\ [0.7, 0.8]\end{pmatrix} & \begin{pmatrix}[0.4, 0.8],\\ [0.5, 0.8]\end{pmatrix} & \begin{pmatrix}[0.3, 0.8],\\ [0.2, 0.2]\end{pmatrix} & \begin{pmatrix}[0.5, 0.8],\\ [0.5, 0.8]\end{pmatrix} \\ S_3 & \begin{pmatrix}[0.8, 0.9],\\ [0.2, 0.3]\end{pmatrix} & \begin{pmatrix}[0.7, 0.9],\\ [0.3, 0.4]\end{pmatrix} & \begin{pmatrix}[0.7, 0.9],\\ [0.1, 0.2]\end{pmatrix} & \begin{pmatrix}[0.8, 0.9],\\ [0.5, 0.6]\end{pmatrix} \\ S_4 & \begin{pmatrix}[0.1, 0.2],\\ [0.3, 0.4]\end{pmatrix} & \begin{pmatrix}[0.2, 0.8],\\ [0.3, 0.3]\end{pmatrix} & \begin{pmatrix}[0.1, 0.9],\\ [0.5, 0.6]\end{pmatrix} & \begin{pmatrix}[0.3, 0.4],\\ [0.3, 0.6]\end{pmatrix} \end{pmatrix}$$

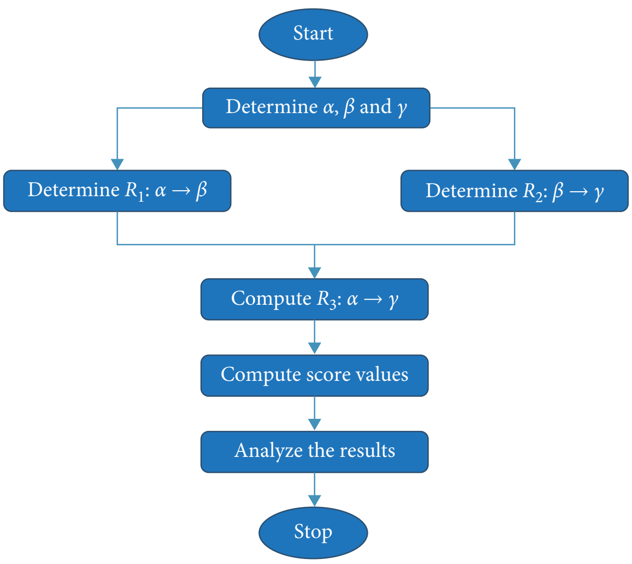

Figure 13: Flowchart describing the proposed medical diagnosis process.

IVTSFNs. All the data is here being in the form of IVTSFNs, where $\alpha = \{\alpha_1 = \text{Amjad}, \alpha_2 = Ali, \alpha_3 = \text{Zeeshan}, \alpha_4 = \text{Naeem}\}$, $\beta = \{\beta_1 = \text{temperature}, \beta_2 = \text{headache}, \beta_3 = \text{cough}, \beta_4 = \text{chest pain}, \beta_5 = \text{flu}\}$, and $\gamma = \{\gamma_1 = \text{malaria}, \gamma_2 = \text{typhoid}, \gamma_3 = \text{allergy}, \gamma_4 = \text{heart problem}, \gamma_5 = \text{viral disease}\}$ denote the sets of patients, symptoms, and diagnosis, respectively. The detailed stepwise demonstration is as follows:

(1) Formation of relation R_1: $\alpha \longrightarrow \beta$ (see Table 5)

(2) Formation of relation R_2: $\beta \longrightarrow \gamma$ (see Table 6)

(3) Computation of R_3: $\alpha \longrightarrow \gamma$ as a composition of R_1 and R_2

(4) Investigation of diagnosis for patients using the score function

The scores of Table 8 represent that α_1 and α_4 , i.e., Amjad and Naeem have allergy problems. Zeeshan having symptoms of malaria and typhoid, while Ali is a patient of malaria too, as indicated by his symptoms.

6.2.1. Comparative Analysis. In this subsection, our aim is to show that if we consider the structures of FSs, IVFSs, IFSs, IVIFSs, PyFSs, IVPyFSs, PFSs, and IVPFS, they all will turn out be a fail case. Conversely, if we have the data in the existing structures, we are able to deal with it using the operations of IVTSFSs. For example, if we convert the data available in Tables 5 and 6 in the form of TSFNs by choosing MGs as a single number from $[0, 1]$ interval instead of closed subinterval of $[0, 1]$, then we will obtain the following relations shown in Table 9:

Now, the data provided in Tables 9 and 10 are purely in the form of TSFNs as described in [6]. This type of data can be shifted into equivalent IVTSFRs of closed subintervals without disturbing the grades.

The information in Tables 9 and 10 is similar to that in Tables 11 and 12. The process of medical diagnosis proposed in an intuitionistic and picture fuzzy environment [43, 51] can also be carried out using the proposed approach. Hence, we can use the tools of IVTSFSs to solve problems of TSFSs and consequently IVPFSs, PFSs, IVIFSs, IFSs, IVFSs, and FSs.

6.3. Shortest Path (SP) Problem. It is one of the important applications of graph theory and got great attention in past decades. A SP from a source to a destination node is required in most of the fields of engineering and other sciences. As briefly explained in the first section of this manuscript, the problem is highly valued in different fuzzy algebraic structures and several new approaches are developed to resolve it. We are going to follow the famous DA for finding an appropriate path problem in the context of IVTSFG. A directed network of finite nodes and edges is considered to find the SP form a source node (SN) to destination node (DN) based on alternative nodes and edges, respectively. The edges denoted the path from one vertex to the adjacent vertex. In real-life, the shortest paths mean a path having low cost to travel on or a path that required less time to reach a destination or a path having the least distance between SN and DN and the information about these parameters is provided in the context of IVTSFNs. The detailed algorithm for finding SP using DA is further described.

6.3.1. Dijkstra Algorithm (DA) for Finding SP in Interval-Valued T-Spherical Fuzzy Environment. DA is a baseline algorithm for the subject problem and for some quality work on fuzzy SP using DA, one may refer to [44–48]. Here, we

Table 5: Interval-valued T-spherical fuzzy relation showing the relation between patients and symptoms.

R_1	β_1	β_2	β_3	β_4	β_5
α_1	$\begin{pmatrix}[0.4, 0.7],\\ [0.1, 0.4],\\ [0.4, 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.5],\\ [0.1, 0.4],\\ [0.7, 0.9]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.7],\\ [0.6, 0.8],\\ [0.5, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.5, 0.8],\\ [0.0, 0.4],\\ [0.4, 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.0, 0.3],\\ [0.7, 0.9],\\ [0.5, 0.8]\end{pmatrix}$
α_2	$\begin{pmatrix}[0.6, 0.9],\\ [0.1, 0.3],\\ [0.0, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.6],\\ [0.0, 0.3],\\ [0.1, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.0, 0.2],\\ [0.3, 0.6],\\ [0.5, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2, 0.4],\\ [0.1, 0.3],\\ [0.2, 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.3, 0.6],\\ [0.4, 0.7],\\ [0.5, 0.9]\end{pmatrix}$
α_3	$\begin{pmatrix}[0.2, 0.6],\\ [0.3, 0.6],\\ [0.3, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.4],\\ [0.0, 0.3],\\ [0.1, 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.5, 0.8],\\ [0.3, 0.6],\\ [0.0, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.0, 0.1],\\ [0.6, 0.9],\\ [0.2, 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.6, 0.8],\\ [0.1, 0.3],\\ [0.2, 0.5]\end{pmatrix}$
α_4	$\begin{pmatrix}[0.0, 0.3],\\ [0.1, 0.3],\\ [0.4, 0.9]\end{pmatrix}$	$\begin{pmatrix}[0.2, 0.7],\\ [0.4, 0.5],\\ [0.3, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2, 0.9],\\ [0.2, 0.6],\\ [0.1, 0.6]\end{pmatrix}$	$\begin{pmatrix}[0.6, 1.0],\\ [0.0, 0.5],\\ [0.0, 0.2]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.5],\\ [0.2, 0.8],\\ [0.5, 1.0]\end{pmatrix}$

Table 6: Interval-valued T-spherical fuzzy relation showing the relation between symptoms and diagnosis.

R_2	γ_1	γ_2	γ_3	γ_4	γ_5
β_1	$\begin{pmatrix}[0.3, 0.6],\\ [0.1, 0.5],\\ [0.2, 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.6],\\ [0.3, 0.7],\\ [0.1, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2, 0.6],\\ [0.0, 0.1],\\ [0.0, 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.6],\\ [0.2, 0.5],\\ [0.7, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.2, 0.6],\\ [0.1, 0.4],\\ [0.4, 0.9]\end{pmatrix}$
β_2	$\begin{pmatrix}[0.6, 0.7],\\ [0.0, 0.3],\\ [0.0, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.0, 0.6],\\ [0.0, 0.3],\\ [0.2, 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.8],\\ [0.1, 0.3],\\ [0.2, 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.7],\\ [0.5, 0.8],\\ [0.3, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.8],\\ [0.2, 0.5],\\ [0.6, 0.9]\end{pmatrix}$
β_3	$\begin{pmatrix}[0.1, 0.5],\\ [0.1, 0.4],\\ [0.4, 0.9]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.2],\\ [0.0, 0.4],\\ [0.3, 0.9]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.5],\\ [0.2, 0.7],\\ [0.3, 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.4, 1.0],\\ [0.0, 0.6],\\ [0.0, 0.3]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.3],\\ [0.2, 0.5],\\ [0.3, 0.7]\end{pmatrix}$
β_4	$\begin{pmatrix}[0.0, 0.4],\\ [0.0, 0.2],\\ [0.4, 1.0]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.6],\\ [0.2, 0.7],\\ [0.0, 0.2]\end{pmatrix}$	$\begin{pmatrix}[0.8, 0.9],\\ [0.0, 0.2],\\ [0.1, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.0, 0.3],\\ [0.3, 0.9],\\ [0.5, 0.9]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.8],\\ [0.1, 0.5],\\ [0.0, 0.4]\end{pmatrix}$
β_5	$\begin{pmatrix}[0.4, 0.7],\\ [0.1, 0.5],\\ [0.3, 0.4]\end{pmatrix}$	$\begin{pmatrix}[0.2, 0.5],\\ [0.1, 0.7],\\ [0.5, 0.7]\end{pmatrix}$	$\begin{pmatrix}[0.4, 0.5],\\ [0.3, 0.7],\\ [0.4, 0.8]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.9],\\ [0.2, 0.4],\\ [0.1, 0.5]\end{pmatrix}$	$\begin{pmatrix}[0.1, 0.7],\\ [0.0, 0.4],\\ [0.6, 1.0]\end{pmatrix}$

discussed the DA for the information provided as IVTSFNs. The IVTSF modified DA [46] (IVTSFDA) is described as follows:

(1) Identify the source as permanent node (P) and assign it the label $([0, 0], [0, 0], [1, 1], -)$ as by default this node is included in low cost path and distance covered at this stage is 0.

(2) For every node j whose path is from node i, compute the label $[v_i \oplus d_{ij}, i]$ if j is not a permanent node. Further, if j is identified as $[v_j, k]$ via some other node, then replace $[v_j, k]$ by $[v_i \oplus d_{ij}, i]$ only if $\mathrm{SC}(v_i \oplus d_{ij})$ is less than $\mathrm{SC}(v_j)$.

(3) The algorithm terminates if all the nodes are permanently labeled. Else, select $[v_r, s]$ having low cost distance v_r and reiterate Step 2 by making $i = r$.

(4) Find the low cost path from SN to DN.

The flowchart is portrayed in Figure 14.

Remarks 2. The label $[v_i \oplus d_{ij}, i]$ means that we reached from node i and covered a distance $v_i \oplus d_{ij}$. Furthermore, it is

Table 7: Composition of two interval-valued T-spherical fuzzy relations in Tables 5 and 6 showing the diagnosis for a patient.

R_3	γ_1	γ_2	γ_3	γ_4	γ_5
α_1	([0.4, 0.6], [0.0, 0.2], [0.4, 0.7])	([0.4, 0.6], [0.0, 0.3], [0.4, 0.6])	([0.5, 0.8], [0.0, 0.1], [0.4, 0.6])	([0.4, 0.7], [0.0, 0.4], [0.5, 0.8])	([0.4, 0.8], [0.0, 0.4], [0.4, 0.6])
α_2	([0.4, 0.6], [0.0, 0.2], [0.1, 0.4])	([0.4, 0.6], [0.0, 0.3], [0.1, 0.5])	([0.4, 0.6], [0.0, 0.1], [0.0, 0.5])	([0.1, 0.6], [0.0, 0.3], [0.3, 0.8])	([0.2, 0.6], [0.0, 0.3], [0.2, 0.5])
α_3	([0.4, 0.7], [0.0, 0.2], [0.1, 0.5])	([0.2, 0.6], [0.0, 0.3], [0.2, 0.5])	([0.4, 0.6], [0.0, 0.1], [0.2, 0.5])	([0.4, 0.8], [0.0, 0.3], [0.0, 0.4])	([0.2, 0.7], [0.0, 0.3], [0.2, 0.5])
α_4	([0.2, 0.7], [0.0, 0.2], [0.3, 0.8])	([0.4, 0.6], [0.0, 0.3], [0.0, 0.2])	([0.6, 0.9], [0.0, 0.1], [0.1, 0.4])	([0.2, 0.9], [0.0, 0.3], [0.1, 0.6])	([0.4, 0.8], [0.0, 0.3], [0.0, 0.4])

Table 8: Score values of the data provided in Table 7.

R	γ_1	γ_2	γ_3	γ_4	γ_5
α_1	0.58	0.58	0.78	0.55	0.56
α_2	0.52	0.48	0.48	0.11	0.28
α_3	0.51	0.28	0.56	0.48	0.29
α_4	0.26	0.50	0.81	0.25	0.48

Table 9: T-spherical fuzzy relation showing the relation between patients and symptoms.

R_1	β_1	β_2	B$_3$	β_4	β_5
α_1	(0.7, 0.1, 0.4	(0.5, 0.1, 0.8)	(0.4, 0.6, 0.5)	(0.8, 0, 0.4)	(0, 0.8, 0.5)
α_2	(0.9, 0.1, 0)	(0.5, 0, 0.2)	(0, 0.6, 0.8)	(0.3, 0.3, 0.3)	(0.5, 0.5, 0.7)
α_3	(0.4, 0.4, 0.4)	(0.2, 0.1, 0.2)	(0.7, 0.4, 0.2)	(0, 0.8, 0.4)	(0.7, 0.3, 0.3)
α_4	(0, 0.1, 0.6)	(0.5, 0.5, 0.5)	(0.9, 0.4, 0.1)	(1, 0, 0)	(0.2, 0.5, 0.7)

Table 10: T-spherical fuzzy relation showing the relation between symptoms and diagnosis.

R_1	γ_1	γ_2	γ_3	γ_4	γ_5
β_1	(0.4, 0.3, 0.4)	(0.5, 0.5, 0.3)	(0.6, 0, 0.1)	(0.4, 0.3, 0.7)	(0.3, 0.3, 0.6)
β_2	(0.7, 0.1, 0.2)	(0, 0, 0.5)	(0.5, 0.1, 0.6)	(0.4, 0.6, 0.6)	(0.1, 0.3, 0.7)
β_3	(0.4, 0.2, 0.7)	(0.2, 0, 0.8)	(0.5, 0.2, 0.4)	(1, 0, 0)	(0.7, 0.4, 0.6)
β_4	(0, 0, 1)	(0.45, 0.4, 0)	(0.9, 0, 0.1)	(0.1, 0.6, 0.77)	(0.7, 0.2, 0.1)
β_5	(0.7, 0.1, 0.4)	(0.41, 0.3, 0.6)	(0.5, 0.5, 0.5)	(0.76, 0.3, 0.2)	(0.2, 0, 0.81)

important to note that we cannot proceed to a permeant node but can go reverse. For two directly connected adjacent nodes i and j respectively, node i is considered as the predecessor of node j if the path connecting them is directed from i to j.

Example 10. Consider Figure 15 where a network of 6 nodes with 8 edges is portrayed. Our aim is to apply modified DA to this network and find the low cost path from SN (N_1) to DN (N_2).

The edges involved in this network are listed in Table 13.

The step-by-step computations of modified DA is as follows:

Step 1. Identify node 1 as permanent due to its lowest cost.

Step 2. The ways directed from node 1 are also possible; i.e., we may either move to node 2 or to node 3. List of nodes is in Table 14.

Now we compute the scores of $([0.1, 0.3], [0.2, 0.6], [0.2, 0.8])$ and $([0.3, 0.5], [0.2, 0.5], [0.5, 0.7])$.

Table 11: Equivalent to Table 9.

R_1	β_1	β_2	β_3	β_4	β_5
α_1	([0.7, 0.7], [0.1, 0.7], [0.4, 0.4])	([0.5, 0.5], [0.1, 0.1], [0.8, 0.8])	([0.4, 0.4], [0.6, 0.6], [0.5, 0.5])	([0.8, 0.8], [0.0, 0.0], [0.4, 0.4])	([0.0, 0.0], [0.8, 0.8], [0.5, 0.5])
α_2	([0.9, 0.9], [0.1, 0.1], [0.0, 0.0])	([0.5, 0.5], [0.0, 0.0], [0.2, 0.2])	([0.0, 0.0], [0.6, 0.6], [0.8, 0.8])	([0.3, 0.3], [0.3, 0.3], [0.3, 0.3])	([0.5, 0.5], [0.5, 0.5], [0.7, 0.7])
α_3	([0.4, 0.4], [0.4, 0.4], [0.4, 0.4])	([0.2, 0.2], [0.1, 0.1], [0.2, 0.2])	([0.7, 0.7], [0.4, 0.4], [0.2, 0.2])	([0.0, 0.0], [0.8, 0.8], [0.4, 0.4])	([0.7, 0.7], [0.3, 0.3], [0.3, 0.3])
α_4	([0.0, 0.0], [0.1, 0.1])	([0.5, 0.5], [0.5, 0.5])	([0.9, 0.9], [0.4, 0.4],)	([1.0, 1.0], [0.0, 0.0],)	([0.2, 0.2], [0.5, 0.5],)

Table 12: Equivalent to Table 10.

R_2	γ_1	γ_2	γ_3	γ_4	γ_5
β_1	([0.4, 0.4], [0.3, 0.3], [0.4, 0.4])	([0.5, 0.5], [0.5, 0.5], [0.3, 0.3])	([0.6, 0.6], [0.0, 0.0], [0.1, 0.1])	([0.4, 0.4], [0.3, 0.3], [0.7, 0.7])	([0.3, 0.3], [0.3, 0.3], [0.6, 0.6])
β_2	([0.7, 0.7], [0.1, 0.1], [0.2, 0.2])	([0.0, 0.0], [0.0, 0.0], [0.5, 0.5])	([0.5, 0.5], [0.1, 0.1], [0.6, 0.6])	([0.4, 0.4], [0.6, 0.6], [0.6, 0.6])	([0.1, 0.1], [0.3, 0.3], [0.7, 0.7])
β_3	([0.4, 0.4], [0.2, 0.2], [0.7, 0.7])	([0.2, 0.2], [0.0, 0.0], [0.8, 0.8])	([0.5, 0.5], [0.2, 0.2], [0.4, 0.4])	([1.0, 1.0], [0.0, 0.0], [0.0, 0.0])	([0.1, 0.1], [0.4, 0.4], [0.6, 0.6])
β_4	([0.0, 0.0], [0.0, 0.0], [1.0, 1.0])	([0.45, 0.45], [0.4, 0.4], [0.0, 0.0])	([0.9, 0.9], [0.0, 0.0], [0.1, 0.1])	([0.1, 0.1], [0.6, 0.6], [0.7, 0.7])	([0.7, 0.7], [0.2, 0.2], [0.1, 0.1])
β_5	([0.7, 0.7], [0.1, 0.1], [0.4, 0.4])	([0.41, 0.41], [0.3, 0.3], [0.6, 0.6])	([0.5, 0.5], [0.5, 0.5], [0.5, 0.5])	([0.76, 0.76], [0.3, 0.3], [0.2, 0.2])	([0.2, 0.2], [0.0, 0.0], [0.81, 0.81])

$$\begin{aligned} SC([0.1,\ 0.3],\ [0.2,\ 0.6],\ [0.2, 0.8]) &= 0.016667, \\ SC([0.3,\ 0.5],\ [0.2,\ 0.5],\ [0.5, 0.7]) &= 0.042967. \end{aligned} \tag{29}$$

As the score of ([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]) is less than ([0.3, 0.5], [0.2, 0.5], [0.5, 0.7]), we mark node 3 as (([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]), N_1) and labeled it permanent.

Step 3. Also, dual routes directed from node 3, i.e., we may either move to node 4 or to node 5. List of nodes is in Table 15.

Now, we compute the scores of ([0.009324, 0.120246], [0.02, 0.24], [0.0, 0.24]) and ([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]) as follows:

$$\begin{aligned} SC([0.009324,\ 0.120246],\ [0.02,\ 0.24],\ [0.0, 0.24]) &= 0.063472, \\ SC([0.000122, 0.019715],\ [0.08,\ 0.48],\ [0.14, 0.72]) &= 0.035482. \end{aligned} \tag{30}$$

As the score of ([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]) is less than ([0.009324, 0.120246], [0.02, 0.24], [0.0, 0.24]), therefore, we mark the node 5 as (([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]), N_3) and labeled it permanent.

Step 4. One way from node 5 exists only, i.e., we can only move to node 6 depicted in Table 16.

As there is only one way from nodes 5 to 6. Therefore, we mark node 6 as (([0.002604, 0.007776], [0.016, 0.24], [0.028, 0.216]), N_5) and labeled it permanent.

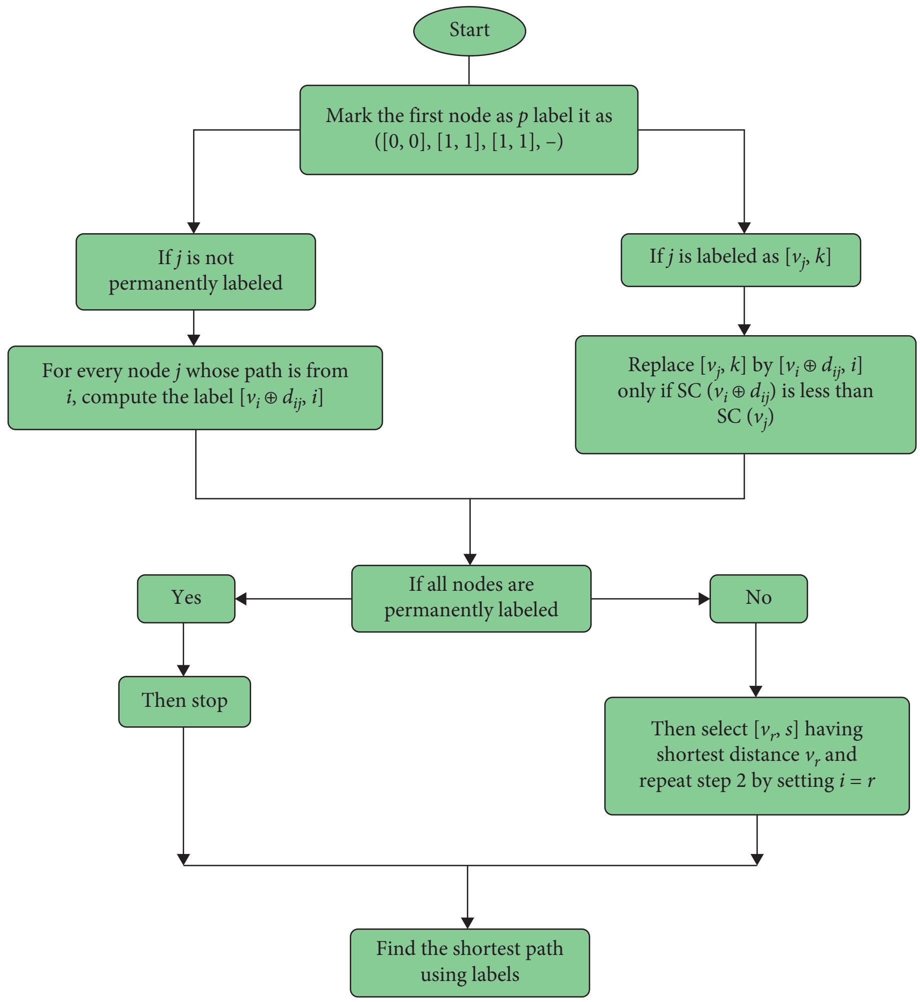

Figure 14: Flowchart of modified DA for computing low cost path.

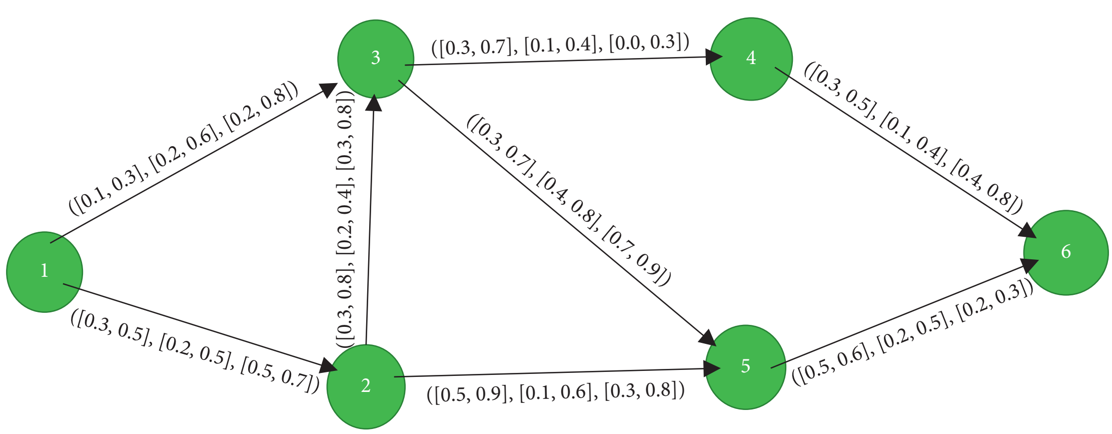

Figure 15: Interval valued T-spherical fuzzy network.

Step 5. Nodes 2 and 4 are the remaining temporary nodes, so their status is changed to permanent and depicted in Table 17.

Step 6. From Table 18, the lowest cost path pattern is from SN to DN, i.e., from nodes 1 to 6.

The SP computed as per modified DA is as follows:

$$N_1 \longrightarrow N_3 \longrightarrow N_5 \longrightarrow N_6. \quad (31)$$

6.3.2. Comparative Study. Consider a network based IVIF environment portrayed in Figure 16. All the path values are

Table 13: Weights of edges.

Edges	Interval-valued T-spherical distances
$(\mathbf{N}_1, \mathbf{N}_2)$	$([0.1, 0.3], [0.2, 0.6], [0.2, 0.8])$
$(\mathbf{N}_1, \mathbf{N}_3)$	$([0.3, 0.5], [0.2, 0.5], [0.5, 0.7])$
$(\mathbf{N}_2, \mathbf{N}_3)$	$([0.3, 0.8], [0.2, 0.4], [0.3, 0.8])$
$(\mathbf{N}_2, \mathbf{N}_5)$	$([0.5, 0.9], [0.1, 0.6], [0.3, 0.8])$
$(\mathbf{N}_3, \mathbf{N}_4)$	$([0.3, 0.7], [0.1, 0.4], [0.0, 0.3])$
$(\mathbf{N}_3, \mathbf{N}_5)$	$([0.3, 0.7], [0.4, 0.8], [0.7, 0.9])$
$(\mathbf{N}_4, \mathbf{N}_6)$	$([0.3, 0.5], [0.1, 0.4], [0.4, 0.8])$
$(\mathbf{N}_5, \mathbf{N}_6)$	$([0.5, 0.6], [0.2, 0.5], [0.2, 0.3])$

Table 14: List of nodes.

Nodes	Label	Status
$\mathbf{N}_1$	$(([0, 0], [0, 0], [1, 1]), -)$	Permanent
$\mathbf{N}_2$	$(([0.3, 0.5], [0.2, 0.5], [0.5, 0.7]), \mathbf{N}_1)$	Temporary
$\mathbf{N}_3$	$(([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]), \mathbf{N}_1)$	Temporary

Table 15: List of nodes.

Nodes	Label	Status
$\mathbf{N}_1$	$(([0, 0], [0, 0], [1, 1]), -)$	Permanent
$\mathbf{N}_2$	$(([0.3, 0.5], [0.2, 0.5], [0.5, 0.7]), \mathbf{N}_1)$	Temporary
$\mathbf{N}_3$	$(([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]), \mathbf{N}_1)$	Permanent
$\mathbf{N}_4$	$(([0.009324, 0.120246], [0.02, 0.24], [0.0, 0.24]), \mathbf{N}_3)$	Temporary
$\mathbf{N}_5$	$(([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]), \mathbf{N}_3)$	Temporary

Table 16: List of nodes.

Nodes	Label	Status
$\mathbf{N}_1$	$(([0, 0], [0, 0], [1, 1]), -)$	Permanent
$\mathbf{N}_2$	$(([0.3, 0.5], [0.2, 0.5], [0.5, 0.7]), \mathbf{N}_1)$	Temporary
$\mathbf{N}_3$	$(([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]), \mathbf{N}_1)$	Permanent
$\mathbf{N}_4$	$(([0.009324, 0.120246], [0.02, 0.24], [0.0, 0.24]), \mathbf{N}_3)$	Temporary
$\mathbf{N}_5$	$(([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]), \mathbf{N}_3)$	Permanent
$\mathbf{N}_6$	$(([0.002604, 0.007776], [0.016, 0.24], [0.028, 0.216]), \mathbf{N}_5)$	Permanent

Table 17: List of nodes.

Nodes	Label	Status
$\mathbf{N}_1$	$(([0, 0], [0, 0], [1, 1]), -)$	Permanent
$\mathbf{N}_2$	$(([0.3, 0.5], [0.2, 0.5], [0.5, 0.7]), \mathbf{N}_1)$	Permanent
$\mathbf{N}_3$	$(([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]), \mathbf{N}_1)$	Permanent
$\mathbf{N}_4$	$(([0.009324, 0.120246], [0.02, 0.24], [0.0, 0.24]), \mathbf{N}_3)$	Permanent
$\mathbf{N}_5$	$(([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]), \mathbf{N}_3)$	Permanent
$\mathbf{N}_6$	$(([0.002604, 0.007776], [0.016, 0.24], [0.028, 0.216]), \mathbf{N}_5)$	Permanent

Table 18: List of nodes.

$\mathbf{N}_6$	$(([0.002604, 0.007776], [0.016, 0.24], [0.028, 0.216]), \mathbf{N}_5)$
$\mathbf{N}_5$	$(([0.000122, 0.019715], [0.08, 0.48], [0.14, 0.72]), \mathbf{N}_3)$
$\mathbf{N}_3$	$(([0.1, 0.3], [0.2, 0.6], [0.2, 0.8]), \mathbf{N}_1)$

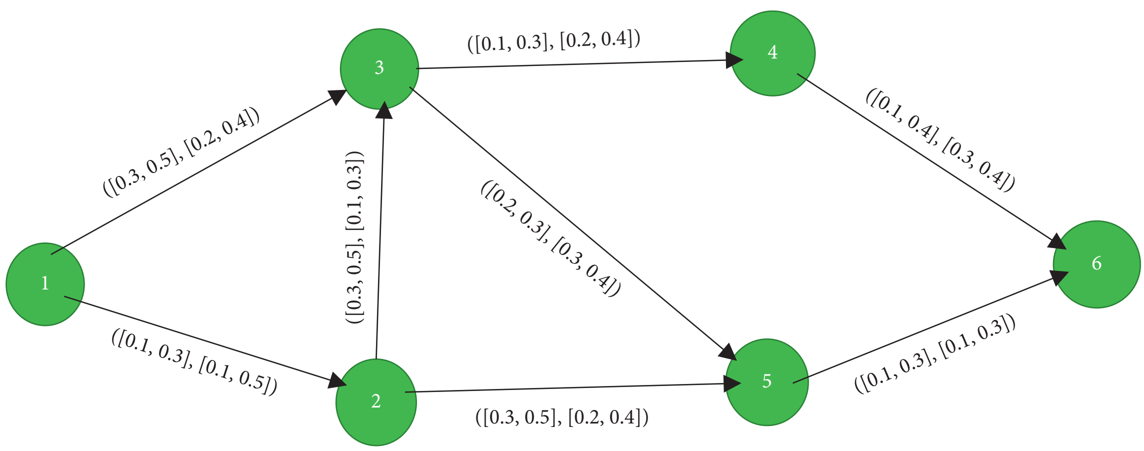

Figure 16: Interval-valued intuitionistic fuzzy network.

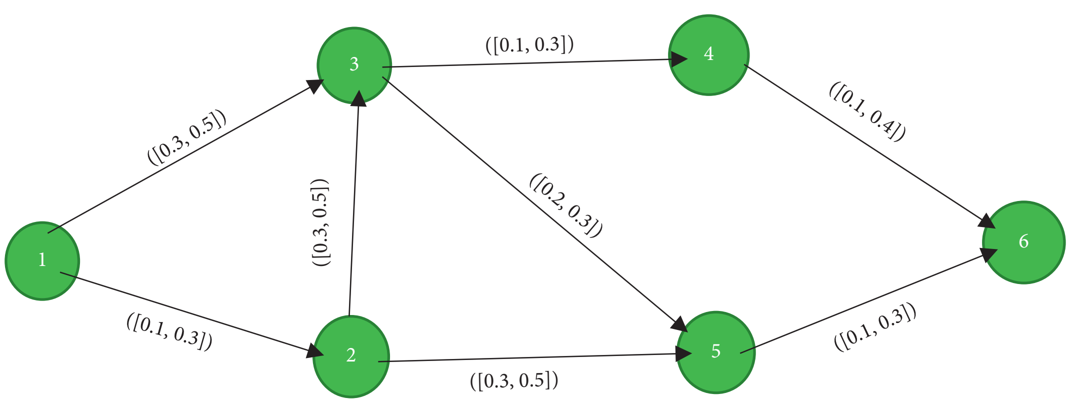

Figure 17: Interval-valued fuzzy network.

in the form of IVIFNs that can be converted into IVTSFNs if we insert the abstinence grade [0.0, 0.0] in it and therefore by using the proposed approach of modified DA, we can find the SP.

Now, consider a network depicted in Figure 17, where all the path values are in the form of IVFNs. As an IVFN can be regarded as IVTSFN, if we insert [0.0, 0.0] in place of abstinence and nonmembership grade. Therefore, using the proposed approach of modified DA, we can solve a low cost path problem.

7. Conclusion

In this manuscript, a new concept of IVTSFSs which generalizes all the existing concepts like FSs, IVFSs, IFSs, IVIFSs, PyFSs, IVPyFS, PFSs, IVPFSs, SFSs, and TSFSs, is proposed. Such a concept is necessary when we have four types of opinion and information is not exact; i.e., information is provided in the form of intervals. Similarly, we developed the concept of IVTSFG which generalizes all the existing definitions of graph theory so far existed. The basic operations and related terms of IVTSFSs and IVTSFGs are defined with their properties. We utilized the aggregation operators of IVTSFSs in anomaly detection problems which is a challenging problem from computer science. We defined relations for IVTSFSs along with their compositions and applied them to solve anomaly detection and medical diagnosis problem. We also proposed a modified DA to solve a low cost path problem in a network based on IVTSFGs. The comparative evaluation demonstrated that the proposed structure is novel and addressed the shortcomings of the existing ideas. In the future, we shall try to develop some aggregation operators for IVTSFSs to deal with a scenario where the other tools fail to be applied.

Data Availability

All data, models, or codes generated or used during the study are available in a repository or online in accordance with funder data retention policies (full citations that include URLs or DOIs are provided).

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

All authors contributed equally to the manuscript.

Acknowledgments

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