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Transportation of Marangoni convection and irregular heat source in entropy optimized dissipative flow

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ABSTRACT

This paper is about entropy generation in Marangoni convective flow due to rotating disk with irregular heat source. Nonlinear mixed convection radiative flow is considered here. Exponential base heat source is addressed. Subsequent equations are transformed to the ODE's by using generalized Karman's variables. Results are found by using NDSolve MATHEMATICA. Through graphical illustration velocity, temperature, Bejan number and entropy generation are discussed. Axial velocity enhances via Marangoni number while temperature decays. Velocity is greater via larger nonlinear thermal convection parameter. Entropy rate boosts up for variation of nonlinear convection parameter and Marangoni number. Bejan number decays against Brinkman number and nonlinear convection parameter.

1. Introduction

Marangoni effect means the mass transfer due gradient of surface tension between two fluids interface. It is also called thermo-capillary convection when it depends on temperature. In 1855 this mechanism is firstly introduce by James Thomson a physicist by "tears if wine." A region where surface tension is high strongly pull the surrounding fluid particles as compared to low surface tension region, so the gradient in the surface tension is the reason why liquid move from high surface tension to low surface tension. Gradient of surface tension is due to temperature or concentration gradient. There are many practical applications of Marangoni effect such as stabilization of soap films, Benard cell or convection cell etc. One of important application is that in production of integrated circuit it is used to dry the silicon wafer after a wet processing otherwise if liquid left on the surface of wafer can damage components due to oxidation. The Marangoni effect is also used in crystal growth, electron beam melting of metals and welding. Gangadharaiah [1] discovered the superposed fluid flow with Marangoni convection. Sreenivasulu et al. [2] discussed the Marangoni convection flow with thermal radiation and Joule heating with heat and mass transfer. Mahanthesh and Gireesha [3,4] analyzed the Marangoni effect on dusty Casson fluid radiative flow with Joule heating and viscous dissipation. Hayat et al. [5] worked on carbon-water nanofluid with Maranogoni convection. More recent studies [5–10] in which Marangoni convection effect is studies in different situations.

Entropy rate in any irreversible process is known as entropy generation or production. There are many reasons due to which entropy generated such as heat exchange, mixing and expanding of substances, motion of bodies, fluid flow, solid deformation, or any irreversible thermodynamic cycle such as thermal machines e.g., heat pumps, air conditioners, heat engines, power plants etc. Due to irreversible processes entropy is generated some of them are (i) Diffusion (ii) Joule heating (iii) Chemical reactions (iv) flow of heat by thermal resistance (v) viscosity of the fluid (vi) friction of solid surfaces. Hayat et al. [11] examined the irreversibility in flow of copper and silver nanoparticles. Goqo et al. [12] analyzed the irreversibility over a porous wedge with thermal radiation and nanofluid. Khan et al. [13] worked on nanomaterial flow of fluid with irreversibility. Some more studies regarding this topic can be seen through refs. [14–22].

This paper is about entropy generation in Marangoni convective flow due to rotation of disk with irregular heat source. Nonlinear mixed convection radiative flow is considered here. Exponential base heat

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source is addressed. Subsequent equations are transformed to the ODE's by using transformations. Graphs are drawn to see the behavior of fluid characteristic via interesting parameters.

2. Statement

Modeling is done for 3D, incompressible, steady Marangoni convective MHD flow due to rotating disk. Heat generation absorption effect is considered in heat equation with irregular heat source. Nonlinear mixed convection flow is considered here. Thermal radiation effect is also considered in heat equation. Exponential base heat source is addressed. Cylindrical model system is shown in Fig. 1. Under all these assumptions governing equations are

$$\frac{\partial u}{\partial r}+\frac{v}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial z^2}-\frac{\sigma^*}{\rho}B_0^2u+g\left[\beta_1(T-T_\infty)+\beta_2(T-T_\infty)^2\right] \tag{2}$$

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k}{(\rho c_p)}\frac{\partial^2 T}{\partial z^2}+\frac{16\sigma^{**}}{3kk^*}\frac{T_\infty^3}{(\rho c_p)}\left(\frac{\partial^2 T}{\partial z^2}\right)+\frac{\mu}{(\rho c_p)}\left(\frac{\partial u}{\partial z}\right)^2+\frac{Q^*}{(\rho c_p)}exp\left(-n\nu_f{}^{-0.5}\Omega^{0.5}z\right)(T-T_\infty) \tag{3}$$

$$\left.\begin{aligned}\mu\frac{\partial u}{\partial z}=\frac{\partial\sigma}{\partial r}=\frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial r},\quad w=0,\quad T=T_0=T_\infty+T_{const}r^2,\quad \text{at } z=0,\\ u\to 0,\quad T\to T_\infty,\ \text{at } z=\infty\end{aligned}\right\} \tag{4}$$

Here $[r, z]$ are cylindrical coordinates, μ, ρ, σ^*, β_1, β_2, g, (u, v, w), σ^{**}, k^*, c_p, T, k, T_∞, n, σ represent dynamic viscosity, density, electrical conductivity, linear and nonlinear thermal convections, gravity, velocity vector, Stefan-Boltzman constant, mean absorption coefficient, specific heat of nanofluid, temperature, thermal conductivity, external flow temperature, exponential constant, temperature constant, surface tension and $\sigma=\sigma_0-\gamma_T(T-T_\infty)$ where γ_T is $\left(-\frac{\partial\sigma}{\partial T}\right)\Big|_{T=T_\infty}$.

Considering

$$u=r\Omega f,\quad v=\sqrt{\nu\Omega}h,\quad \theta(\xi)=\frac{T-T_\infty}{Ar^2},\quad \xi=z\sqrt{\frac{\Omega}{\nu}} \tag{5}$$

We arrive

$$2f+h'=0 \tag{6}$$

$$f''-hf'-f'^2-Mf'+\lambda\theta(1+\beta_t\theta)=0 \tag{7}$$

$$\frac{1}{Pr}(1+R)\theta''-2f\theta-h\theta'+QExp[-n\xi]+Ecf'^2=0 \tag{8}$$

$$h(0)=0,\ f'(0)=-2Ma,\ \theta(0)=1,\ f'(\infty)=0,\ \theta(\infty)=0 \tag{9}$$

Note that λ, Ma, M, R, Q, E_c, β_i, and Pr highlight mixed convection parameter, Marangoni number, Hartmann number, radiation parameter, heat generation parameter, Eckert number, nonlinear thermal convection parameter and Prandtl number.

Mathematically these variables are

$$\left.\begin{aligned}\lambda\left(=\frac{g\beta_1(T_0-T_\infty)}{r\Omega^2}\right),\ Ma\left(=\frac{\gamma_T Ax}{\mu\Omega}\right),\ Pr\left(=\frac{\nu_f}{\alpha_f}\right),\ M\left(=\frac{\sigma B_0^2}{\rho\Omega}\right),\\ R\left(=\frac{16\sigma^{**}T_\infty^3}{3kk^*}\right),\ Q\left(=\frac{Q^*}{(\rho c_p)\Omega}\right),\ Ec\left(=\frac{(r\Omega)^2}{c_{p1}(T_0-T_\infty)}\right)\\ ,\ \beta_t\left(=\frac{\beta_2(T_0-T_\infty)}{\beta_1}\right)\end{aligned}\right\} \tag{10}$$

Mathematically, entropy generation in presence of thermal radiation and Joule heating is addressed as

$$S_G=\frac{k}{T_\infty^2}\left(1+\frac{16\sigma^*T_\infty^3}{3k^*k}\right)\left(\frac{\partial T}{\partial z}\right)^2+\frac{\mu}{T_\infty}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma^*B_0^2}{T_\infty}u^2 \tag{11}$$

dimensionless form is

$$N_G=(1+R)\theta'^2+Brf'^2+BrMf^2 \tag{12}$$

The Bejan number which is the ratio of heat transport to total entropy is

$$Be=\frac{(1+R)\theta'^2}{Brf'^2+BrMf^2} \tag{13}$$

where $Br\left(=\frac{\mu(r\Omega)^2T_\infty}{kA^2r^4}\right)$ and $N_G\left(=\frac{S_GT_\infty^2\nu}{kA^2r^4a}\right)$ signify the Brinkman number and entropy generation rate.

3. Physical description

This section is dedicated to analyze the trends of velocity, temperature, entropy generation and Bejan number via involved pertinent parameters. For this purpose Figs. (1–14) are constructed.

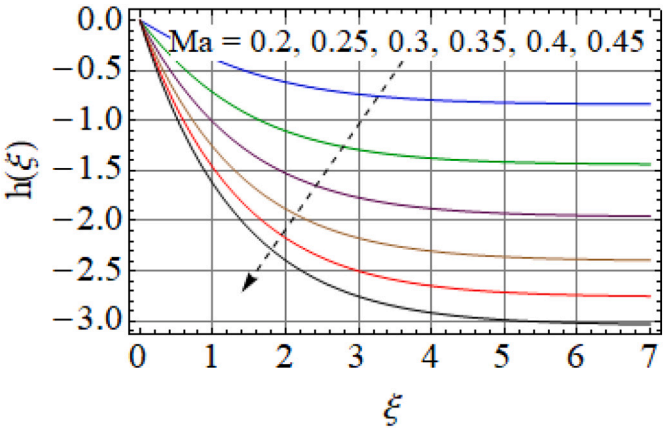

Fig. 1. $h(\xi)$ versus Ma.

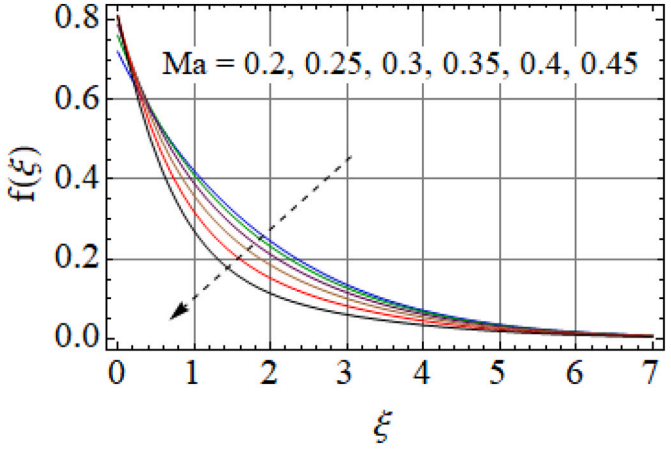

Fig. 2. $f(\xi)$ versus Ma.

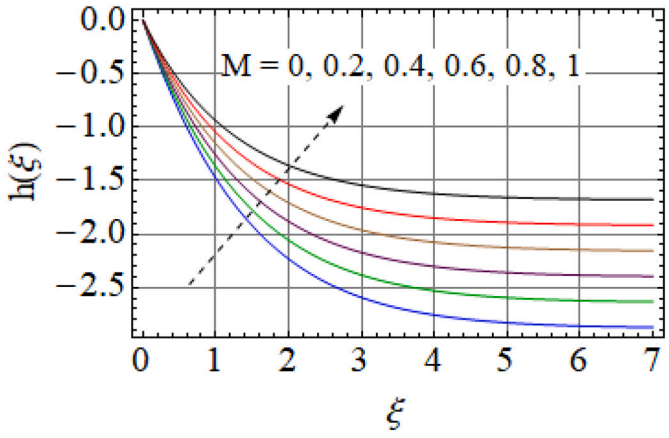

Fig. 3. $h(\xi)$ versus M.

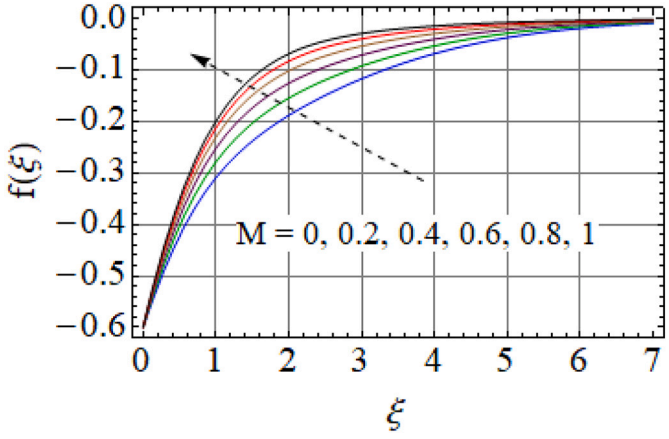

Fig. 4. $f(\xi)$ versus M.

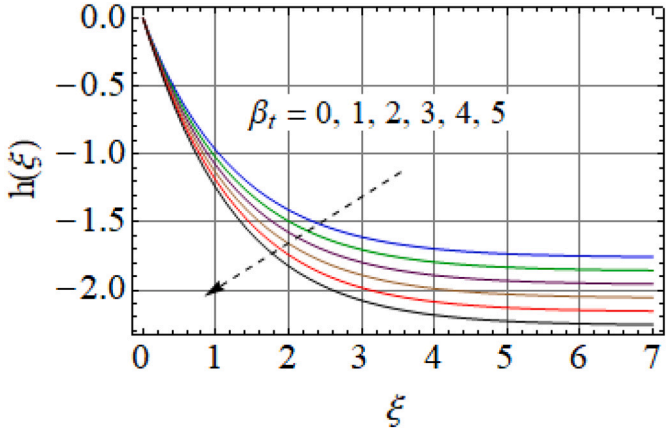

Fig. 5. $h(\xi)$ versus β_t.

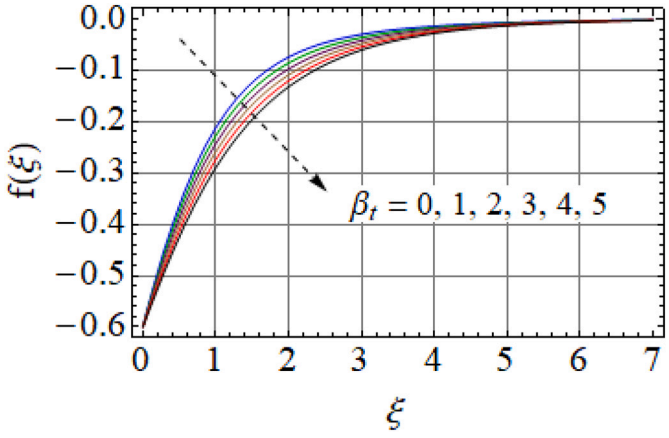

Fig. 6. $f(\xi)$ versus β_t.

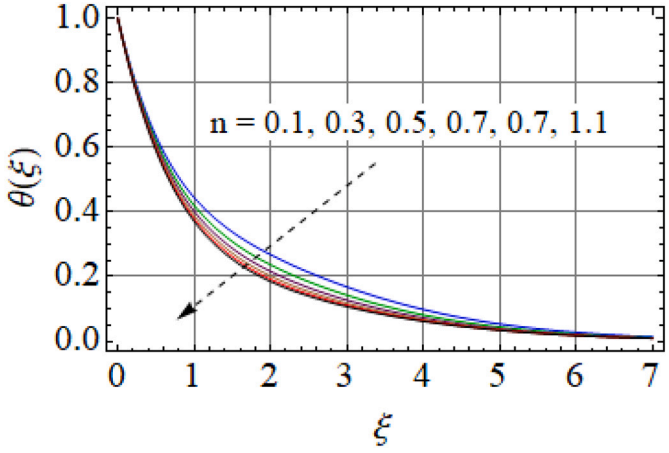

Fig. 7. $\theta(\xi)$ versus n.

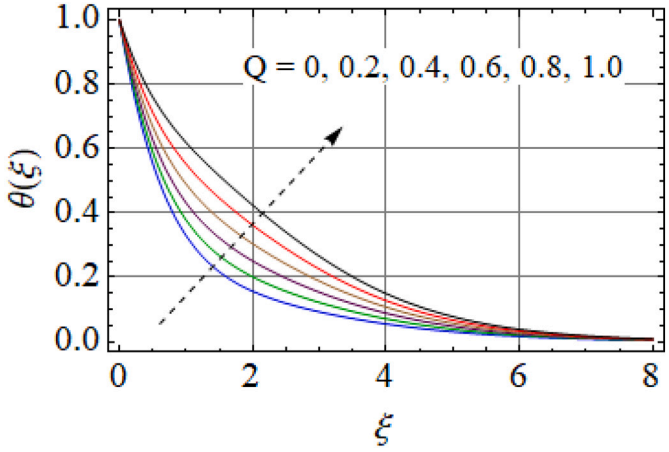

Fig. 8. $\theta(\xi)$ versus Q.

3.1. Velocity

For detail analysis of velocity profiles ($h(\xi)$, $f(\xi)$) against pertinent parameters Figs. (1–6) are constructed. Figs. 1 and 2 deal with the effect of Marangoni number against axial and radial velocity profiles. Axial velocity ($f(\xi)$) enhances for higher variation of Marangoni number (Ma = 0.2, 0.25, 0.3, 0.35, 0.4, 0.45) while radial velocity decreases. With increase in (Ma) surface tension increases due to which momentum boundary layer thickness and axial velocity rises. Figs. 3 and 4 demonstrate the features of axial and radial velocity profile against Hartmann number. It is seen that magnitude of axial and radial velocity profile decays for higher estimation of (M) Hartmann number is a increasing function of Lorentz force. With increase in (M) Lorentz force increases due to which resistance between the fluid particles enhances hence velocity decays for both axial and radial direction. Figs. 5 and 6 examine the trend of axial and radial velocity profile against nonlinear thermal convection parameter (β_t). Here ($h(\xi)$, $f(\xi)$) rises for greater

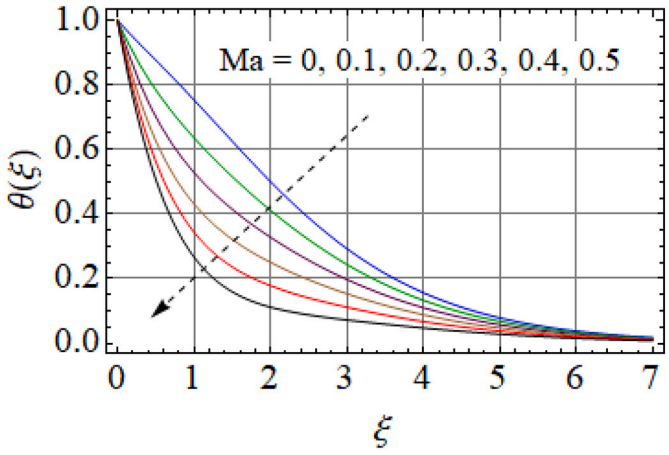

Fig. 9. $\theta(\xi)$ versus *Ma*.

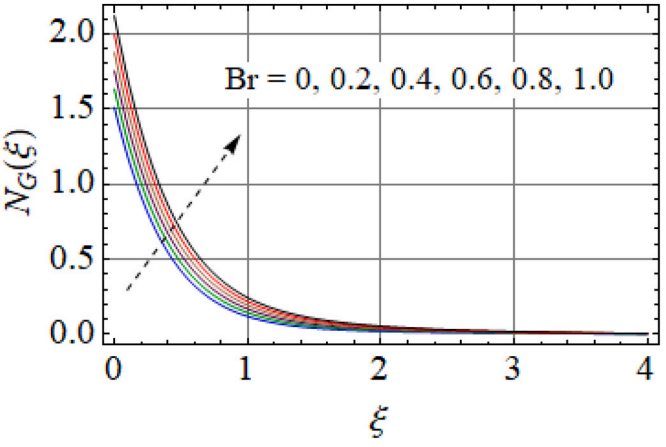

Fig. 12. $N_G(\xi)$ *versus Br*.

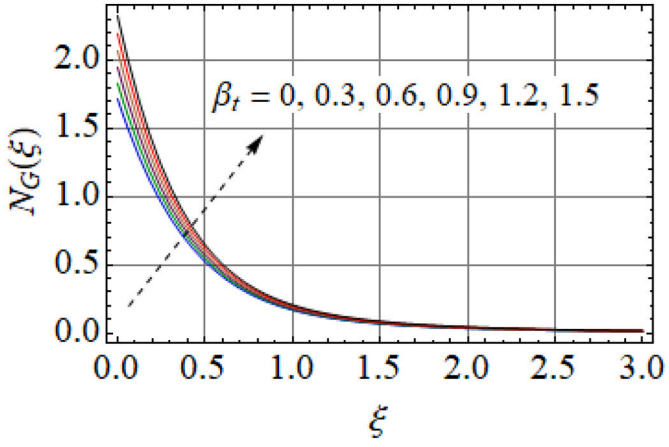

Fig. 10. $N_G(\xi)$ *versus* β_t.

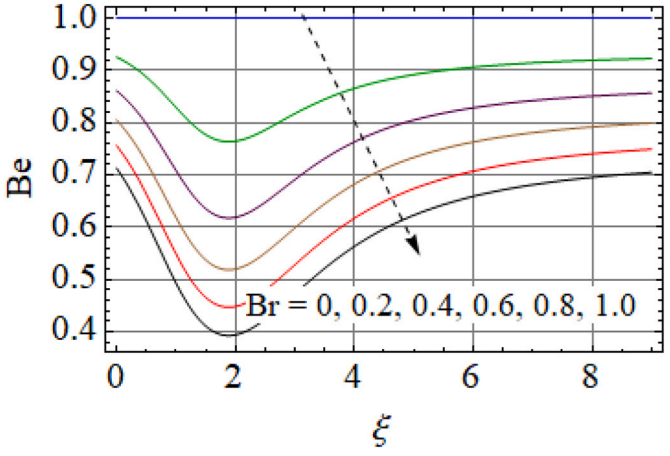

Fig. 13. *Be versus Br*.

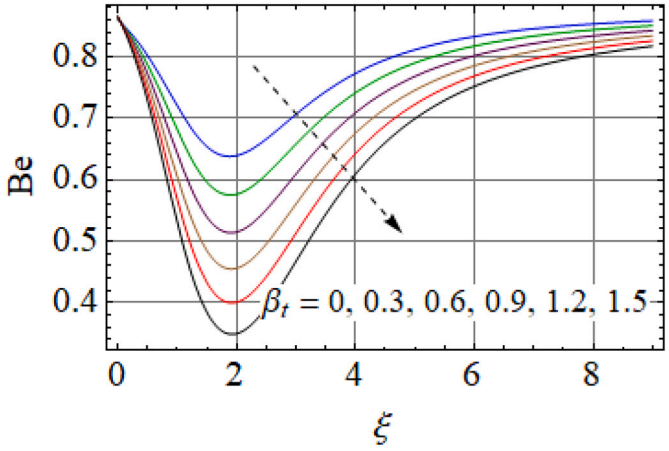

Fig. 11. *Be versus* β_t.

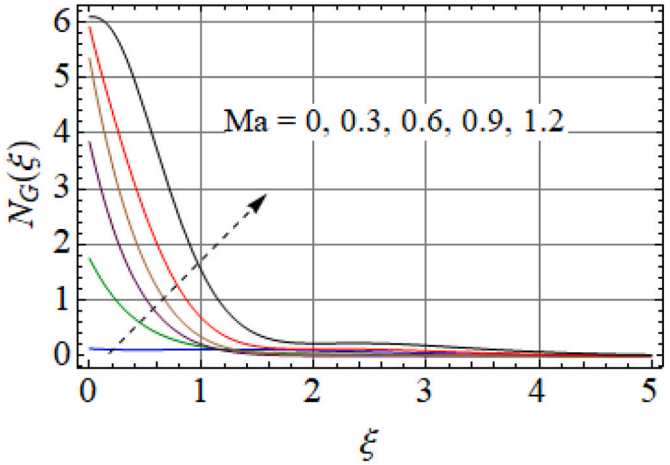

Fig. 14. $N_G(\xi)$ *versus Ma*.

estimations of (β_t). With increase in values of (β_t) temperature difference between disk and ambient rises due to which velocity boosts up.

3.2. Temperature

Impact of exponential index (n), heat generation parameter (Q) and Marangoni number (Ma) on temperature field is seen in Figs. (7–9). Here temperature of the fluid decay with increment of (n) because n is negative power of exponential due to which heat generation effect starts decreasing drastically. Fig. 8 describes the variation of ($\theta(\xi)$) against heat generation parameter (Q). Temperature of the fluid enhances via larger (Q). Marangoni number effect on temperature is depicted in Fig. 9. Temperature of the fluid decays for higher *Ma*. With increase in (Ma) surface tension increases it means that temperature is decreasing

and strong force of attraction exist between surface and molecules.

3.3. Entropy and Bejan number

Figs. 10 and 11 delineate the impact of nonlinear thermal convection parameter (β_t) on entropy generation and Bejan number (*Be*).Here one can see that entropy of the system is increasing with respect to (β_t). It is due to the fact that temperature difference rises with increase in (β_t) due to which total entropy of the system enhances (See Fig. 10). On the other hand Bejan number decays for higher estimation of (β_t). It means that heat transfer irreversibility effect are less that viscous dissipation irreversibility effect (See Fig. 11). Figs. 12 and 13 describe the influence of Brinkman number (*Br*) against entropy generation and Bejan number. It is due to the fact that viscosity is in direct relation with (*Br*) so when viscosity of the fluid enhances resistance produces due to which disorderedness in the system increases (See Fig. 12). Fig. 13 show that Bejan number decays for larger values of (*Br*). It shows that viscous dissipation irreversibility is prominent over heat transfer irreversibility (See Fig. 14). Fig. 14 shows the impact of Marangoni number (*Ma*) against entropy generation. Here we can see that entropy of the fluid rises due to increase in (*Ma*)..

4. Conclusions

Important results of present study are mentioned below:

- Axial and radial velocities show increasing trend via Marangoni number and nonlinear thermal mixed convection parameter.
- Temperature decays via *Q* and *n*.
- Entropy generation boosts up for greater *Br*, β_t and *Ma*.
- Bejan number decays for *Br* and β_t.

Data availability

The data that support the findings of this study are available within the article, the data are made by the authors themselves and do not involve references of others.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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