Surfaces and Interfaces 22 (2021) 100849

Contents lists available at ScienceDirect

Surfaces and Interfaces

journal homepage: www.sciencedirect.com/journal/surfaces-and-interfaces

Significance of temperature-dependent viscosity and thermal conductivity of Walter's B nanoliquid when sinusodal wall and motile microorganisms density are significant

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ARTICLE INFO

Keywords:
Walter's B nanoliquid
Motile microorganisms density
Sinusodal wall
Temperature-dependent viscosity

ABSTRACT

The progressed thermal features of nanoparticles in presence of Lorentz force, chemical reaction and activation energy utilizes novel applications various engineering and industrial processes, thermal engineering and medical sciences. On this end, current continuation aims to address the temperature dependent viscosity and variable thermal consequences in order to inspect the bioconvection aspects in non-Newtonian nanofluid over periodically moving surface. The rheological consequences for non-Newtonian fluid are addressed by using Walter's B liquid. The activation energy effects are also tested to improve the concentration of nano-materials more effectively. The Brownian motion and thermophoretic diffusion significances are incorporated explicitly with alliance of Buongiorno's mathematical model. Although some studies are already reported for nanofluid with constant viscosity but consideration of temperature dependent viscosity for bioconvection of non-Newtonian nanoparticles still need to be attention. Unlike typical contributions, here the flow is originated by periodically accelerated configuration. The appropriate variables enable to develop the non-dimensional for which the employed analytical simulations are carried out via homotopic procedure. The flow parameters are physically inspected with applications of various curves. The numerical iterative values are compiled for local Nusselt number, local motile density number and local Sherwood expressions. The results reveal that a reduced velocity profile is augmented to be the viscosity parameter. The assumption of variable thermal conductivity and temperature dependent viscosity are more effective for enrichment of nanoparticles temperature. It is also concluded that the microorganism for nanofluid increases for buoyancy ratio constant and bioconvection Rayleigh number.

1. Introduction

Many investigations on the characteristics of heat transfer are being performed by engineers and researchers as this phenomenon preserve diverse engineering and industrial significances. Few of such applications of this phenomenon include thermal extrusion processes, various devices heating, energy production, food industries, chemical processing, nano-electronics, space heating, power generation, refrigeration etc. In heat transfer process, the thermal conductivity is assumed to be most important parameter which is considered as constant in traditional heat transportation problems. However, based on several theoretical and experimental observations, it is noted that the thermal conductivity is intimately associated with the deviation of temperature (Animasaun [1,2], Khan and Shehzad [3]). Due to enormous evolution in the thermal engineering and nano-technology specially in last two decades, the dynamic of nano-materials become the novel interest of many engineers

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https://doi.org/10.1016/j.surfin.2020.100849
Received 6 September 2020; Received in revised form 27 October 2020; Accepted 22 November 2020
Available online 27 November 2020

and scientists. Based on various traditional energy recourses, the low energy production is always noticed due to poor thermal performances of involved materials. Recent research in nano-technology proves that blending of nano-sized solid particles with base materials result excellent thermal performances. Nanofluid is the base fluid and solid nano-particles amalgamation with enhanced thermal properties. In fact, the nanofluids belong to the modern era of nano-technology which encounter multidisciplinary engineering, industrial and bio-medical applications. In the field of thermal engineering, the nanoparticles are used to improve the cooling process, extrusion processes and metallurgical applications. The leading continuation on nanofluid was pioneered by Choi [4] in 1995 which provides a direction to researchers to perform more work on this topic. Boungiorno [5] determine the thermophoretic and Brownian motion aspects of nanofluid which is associated with the flow mechanisms. Turkyilmazoglu [6] elaborated the heat transfer mechanisms in plane wall jet with applications of nanoparticles. Waqas and co-investigators [7] inspected the nanofluid properties due to rotating disk in presence of several slip factors. The nanofluid flow for rate type material (Maxwell fluid) with activation energy impact has been analytically worked out by Ijaz and Ayub [8]. Nadeem et al. [9] examine the slip investigation for oscillatory moving surface where micropolar nanoparticles have been immersed uniformly. The thermal mechanisms for peristaltic flow of Jaffrey nanofluid additionally impacted with activation energy features were led by Hayat et al. [10]. Khan and Shehzad [11] discussed the familiar Brownian motion and thermophoretic consequences of nanofluid for third grade liquid possessed by oscillatory surface. Ibrahim [12] tested the higher order slip consequences for Williamson nanofluid flow under magnetic force influence. Waqas and co-researchers [13] scrutinized the joint typical consequences in Maxwell nano-material and micropolar nanoliquid confined by a porous space. Shahzad et al. [14] discussed the Joule heating, chemical reaction and dissipation effects for magnetized tangent hyperbolic nano liquid numerically by employing the shooting algorithm. Ibrahim et al. [15] used modified Fick and Fourier's expressions for convective thermally developed flow of tangent hyperbolic material over a moving surface. Turkyilmazoglu [16] reported most interesting flow mechanisms of single phase nanofluid flow and performing the stability analysis.

Owing to the nonlinear rheological dynamics of non-Newtonian materials, a multidisciplinary work has been done by researchers on this topic. The complex nature of these non-Newtonian fluids provides stimulating challenges for scientists to explore more physical distinct characteristics. It is well justified fact that the mechanical behavior of various complex materials cannot be completely explained by suing traditional viscous liquids. The applications of non-Newtonian materials become more diligent. Some prime and daily routine applications of non-Newtonian fluids involves blood, lava, mayonnaise, biological liquids, butter, synovial liquids and many more. These materials convey novel significances in various industrial processes, food industries, biomedical applications, chemical processes and processing industries. Due to highly disserted nature of such liquids, scientists have signifies different non-Newtonian fluid models upon diverse rheological characteristics. Amongst diverse non-Newtonian models, Walter's-B fluid model is one which has been extensively attended the researcher's attention. This model belong differential type of non-Newtonian materials. The basic investigation of Walter's B liquid was concentrated by Beard and Walter [17]. The fluid model successfully attributes the complex tribological liquids, biotechnological and polymeric materials [18,19].

The bioconvection phenomenon involves the convection motion of liquids at microscopic level. This interesting phenomenon is attributed to the density gradient which yields form microorganisms swimming. These microorganisms always concentrate in upper region of fluid layer and subsequently the upper surface becomes more densers when compared with lower fluid surface. Such differentiation between both regions simulates the instability and as a results different flow trends appeared within the system. The bioconvection phenomenon utilizes bio-technology and bio-engineering applications like enzyme biosensors, bio-fuels etc. In microbial-enhanced oil recovery, the nutrients and various microorganisms immersed in surface of oil bearing to improve the permeability variation. The bioconvection system has been specified different types of microorganisms like gyrotactic, chemotaxis, and geotactic microorganisms. Beside this, the theory of bioconvection of nanofluids elaborates some wide range physical applications like automotive coolants, building designs, medical suspensions sterilization applications, nano-material processing and polymer coating. Kuznetsov [20,21] formulated the flow problem regarding the bioconvection of nanoparticles which was later on, focused by many investigators. Uddin et al. [22] investigated the multiple slip consequences and Stefan blowing aspects for nano-materials in presence of gyrotactic microorganisms confined by porous space. Saini and Sharma [23] focused on the thermo-bioconvection for nanoparticles in saturated porous space by confirming numerical simulations. The gyrotactic microorganisms with viscous nanofluid altered by a truncated cone were directed by Khan et al. [24]. Farooq et al. [25] determine the numerical solution by following shooting procedure for a bioconvection flow of Sisko nanofluid in presence of mixed convection and by imposing higher order slip features. Zhao et al. [26] successfully performed the stability analysis for flow of nano-materials with gyrotactic microorganisms. Waqas et al. [27] reported the Falkner-Skan flow of nanofluid with gyrotactic microorganisms and thermal radiation. The study of Oldroyd-B nanofluid with gyrotactic microorganisms over accelerating surface has been visualized analytically by Khan et al. [28]. Lu et al. [29] used the anisotropic slip effects along with activation energy for three-dimensional flow of nanofluid with gyrotactic microorganisms over a moving stretched surface. Khan et al. [30] examined the transient flow of Oldroyd-B nanofluid with motile microorganism over a radiative stretched surface. The bioconvection slip flow of Eyring-Powell nanofluid under the influence of activation energy over a bidirectional stretched surface was intended by Khan et al. [31]. Ferdows et al. [32] examined the thermal performances of dissipative nanofluid with motile microorganisms configured by an exponential stretched surface. The flow of nano-materials with gyrotactic microorganisms additionally impacted with thermal radiation and activation energy consequences was explored by Zhang et al. [33]. Khan et al. [34] analyzed the bioconvection assessment in thixotropic nano-material over stretched surface. Tlili et al. [35] reported the double stratification slip flow of micropolar nanofluid with gyrotactic microorganisms analytically. Motsa and Animasaun [36] explored the unsteady flow of nano-materials with gyrotactic microorganisms over convectively heated surface with help of passive controlled conditions. Sivaraj et al. [37] utilized the Hall features in bioconvection flow of CuO-water nanoparticles. The magnetized flow of Jeffrey nanofluid with gyrotactic microorganisms induced by a vertical cone was directed by Saleem et al. [38].

Yet, no investigators delineated the temperature dependent viscosity, activation energy and variable thermal conductivity features in bioconvection flow of non-Newtonian fluids induced by periodically accelerating surface. The rheological consequences of non-Newtonian fluids are determined by utilizing Walter's B liquid. The flow induced by a periodically moving surface was originally directed by Wang [39] which was further extended by many investigators. For example Zheng et al. [40] discussed Soret and Dufour effects in flow of viscous fluid induced by a stretched surface with sinusodal wall. The numerical work of Abbas et al. [41] conveys the flow of second grade fluid configured by an accelerated surface. Khan et al. [42] analyzed the bio-convective flow of Jeffrey nanofluid over oscillatory stretching surface. The novel motivations and distinguish features for current work is summarized as:

Ø Analyze the heat and mass transfer characteristics in bioconvection flow of non-Newtonian fluid.

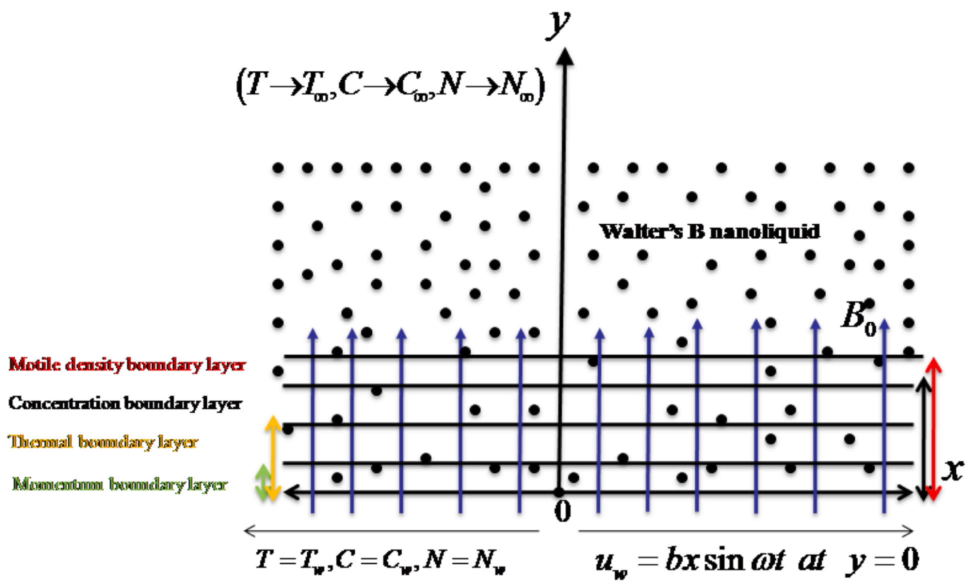

Fig. 1. Geometry of flow problem.

- Ø The distinguish features of non-Newtonian materials has been predicted by using famous Walter's B liquid.
- Ø The viscosity of fluid is assumed to be temperature dependent.
- Ø The variable thermal conductivity is considered for enhancement of nanofluid temperature.
- Ø The concentration of non-Newtonian nanofluid has been observed by using activation energy consequences.
- Ø The flow due to accelerated surfaces encountered special attention due to interesting significance like oscillating jet, fluidic oscillators, oscillation problems, aerodynamics etc.
- Ø The flow has been assumed as unsteady for which modeling of flow problem yield nonlinear partial differential equations which are solved via convergent technique. A comprehensive analysis for involved flow parameters based on different graphs is performed with physical justifications.

2. Mathematical modeling

Let us model the flow equations by using various fundamental laws. The Walter's B nanoliquid with gyrotactic microorganisms has been assumed over periodically accelerated surface under the influence of magnetic force. The Arrhenius energy relations are followed to impose the activation energy features while the characteristics of Brownian motion and thermophoretic aspects are convoluted by using Buongiorno's mathematical nanofluid. Since the flow is assumed as two-dimensional so the velocity component u is taken in $x-$ direction while in $y-$ axis, the velocity component is denoted by v, as shown in Fig. 1. The other physical quantities like nanoparticles temperature, concentration and motile microorganisms density are is expressed by T, C and N, respectively. In current flow modeling the viscosity is temperature while thermal conductivity is of variable nature. Let T_w, C_w, N_w, T_∞, C_∞ and N_∞ symbolizes the surface temperature, surface concentration, surface motile density, free stream nanoparticles temperature, free stream concentration and fee stream motile density, respectively. The flow equations with certain flow assumptions are (Wang - [39] and Zheng et al. [40]):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho_f}\frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right)-\frac{\Omega_0}{\rho_f}\left[\frac{\partial^3 u}{\partial t\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right]-\left(\frac{\sigma_m B_0^2}{\rho_f}\right)u+\frac{1}{\rho_f}\left[(1-C_\infty)\rho_f\beta^* g(T-T_\infty)-(\rho_p-\rho_f)g(C-C_\infty)-(N-N_\infty)g\gamma^*(\rho_m-\rho_f)\right], \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{(\rho c_p)_f}\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)+\frac{(\rho c_p)_p}{(\rho c_p)_f}\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}, \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K_r^2(C-C_\infty)\left(\frac{T}{T_\infty}\right)^n exp\left(\frac{-E_a}{\kappa T}\right), \tag{4}$$

$$\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}+\frac{bW_c}{(C_w-C_\infty)}\left[\frac{\partial}{\partial y}\left(N\frac{\partial C}{\partial y}\right)\right]=D_m\left(\frac{\partial^2 N}{\partial y^2}\right), \tag{5}$$

Since in current attempts, the viscosity is temperature so we use the following Reynolds number exponential equation to modify Eq. (2):

$$\mu(\theta)=e^{-(\alpha\theta)}=1-(\alpha\theta)+O(\alpha^2), \tag{6}$$

where α is viscosity vector.

Now for variable thermal conductivity, we suggest following law [47]:

$$k(T)=k_\infty\left[1+\varepsilon\frac{(T-T_\infty)}{\Delta T}\right], \tag{7}$$

where ε is conductivity constant while k_∞ symbolize theambient fluid conductivity.

Following boundary assumptions for the formulated set of equations are imposed (Wang [39] and Zheng et al. [40]):

$$u=u_\omega=bx\sin\varpi t,\, v=0,\, T=T_w,\, C=C_w,\, N=N_w \text{at} \quad y=0,\, t>0, \tag{8}$$

$$u\to 0,\, \frac{\partial u}{\partial y}\to 0,\, T\to T_\infty,\, C\to C_\infty,\, N\to N_\infty \text{at} \quad y\to\infty. \tag{9}$$

The physical quantities are time (t), fluid density (ρ_f), Walter's B liquid parameter (Ω_0), magnetic field strength (B_0), gravity (g), kinematic viscosity (ν), electrical conductivity (σ_m), nanoparticles density (ρ_p), rate constant (n), volume suspension coefficient (β^*), motile microorganism particles density (ρ_m), microorganisms diffusion constant (D_m), thermal conductivity (k), diffusion constant (D_B), Boltzmann constant κ, chemotaxis constant (b_1), swimming cells speed (W_e), activation energy (E_a), reaction rate (K_r) and Boltzmann constant (κ).

Following variables are incorporated to retained the dimensionless form of flow equations [40]:

$$\eta = \sqrt{\frac{b}{\nu}}y, \tau = t\omega, u = bxf_y(\eta,\tau), v = -\sqrt{\nu b}f(\eta,\tau), \tag{8a}$$

$$\theta(\eta,\tau) = \frac{T-T_\infty}{T_w-T_\infty}, \varphi(\eta,\tau) = \frac{C-C_\infty}{C_w-C_\infty}, \psi(\eta,\tau) = \frac{N-N_\infty}{N_w-N_\infty}, \tag{9a}$$

After invoking the above relations in governing equations, one may achieve

$$(1-(\alpha\theta))f_{\eta\eta\eta} - \alpha\theta_\eta f_{\eta\eta} - Sf_{\eta\tau} - f_\eta^2 + ff_{\eta\eta} - Haf_\eta - K\left(Sf_{\eta\eta\eta\tau} + 2f_\eta f_{\eta\eta} - f_{\eta\eta}^2 - ff_{\eta\eta\eta\eta}\right) + \lambda(\theta - N_\alpha\varphi - N_\beta\chi) = 0, \tag{10}$$

$$(1+\varepsilon\theta)\theta_{\eta\eta} + \varepsilon(\theta_\eta)^2 + \Pr\left[-S\phi_\tau + f\phi_\eta + Nb\theta_\eta\phi_\eta + Nt(\theta_\eta)^2 + \delta\theta\right] = 0, \tag{11}$$

$$\varphi_{\eta\eta} + \frac{Nt}{Nb}\theta_{\eta\eta} - S(Sc)\varphi_\tau + Scf\varphi_\eta - (Sc)\sigma(1+\delta\theta)^n\exp\left(\frac{-E}{(1+\delta\theta)}\right)\varphi = 0, \tag{12}$$

$$\psi_{\eta\eta} - S(Lb)\psi_\tau + Lb\psi_\eta - Pe[\varphi_{\eta\eta}(\chi+\Omega) + \psi_\eta\varphi_\eta] = 0, \tag{13}$$

The boundary conditions are transformed form are

$$f_\eta(0,\tau) = \sin\tau, f(0,\tau) = 0, \theta(0,\tau) = 1, \varphi(0,\tau) = 1, \psi(0,\tau) = 1, \tag{14}$$

$$f_\eta(\infty,\tau)\to 0, \quad \theta(\infty,\tau)\to 0, \varphi(\infty,\tau)\to 0, \psi(\infty,\tau)\to 0, \tag{15}$$

where $K = b\Omega_0/\nu\rho_f$ is viscoelastic parameter, $S = \varpi/b$ oscillating frequency to stretching rate ratio, $M = \sigma B_0^2/\rho_f b$ is Hartmann number, $\lambda = \beta^* g(1-C_\infty)(T_w-T_\infty)/b^2$xmixed convection parameter, $N_\alpha = (\rho_p-\rho_f)(C_w-C_\infty)/\beta^*\rho_f(1-C_\infty)T_\infty$ buoyancy ratio constant, $N_\beta = \gamma^*(n_w-n_\infty)(\rho_m-\rho_f)/\beta^*\rho_f(1-C_\infty)(T_w-T_\infty)$ bioconvection Rayleigh number, $\Pr = \nu/\alpha_f$Prandtl number, variable thermal conductivity parameter $\varepsilon\frac{n!}{r!(n-r)!}$, $Nb = \Gamma_1 D_B(C_w-C_\infty)/\nu$ is Brownian motion parameter, $Nt = \Gamma_1 D_T(T_w-T_\infty)/T_\infty\nu$ is thermophoresis parameter, reaction constant $\sigma = \frac{K_r^2}{c}$, $Pe = b_1 w_c/D_m$ Peclet number, $E = \frac{E^*}{kT_\infty}$ is the activation energy constant, $\Omega = n_\infty/(N^*{}_w - N^*_\infty)$. microorganisms concentration difference constant, while $Lb = \nu/D_m$ define bioconvection Lewis number.

We propose the relations for local Nusselt number, local Sherwood number and motile density number by using following mathematical expressions [47]:

$$\left.\begin{aligned} Nu_x &= \frac{xq_h}{k(T_w-T_\infty)}, q_h = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \\ Sh_x &= \frac{xq_s}{D_B(C_w-C_\infty)}, q_s = -D_B\left(\frac{\partial C}{\partial y}\right)_{y=0}, \\ Nn_x &= \frac{xq_n}{D_m(N_w-N_\infty)}, q_n = -D_m\left(\frac{\partial N}{\partial y}\right)_{y=0}, \end{aligned}\right\} \tag{16}$$

with mass flux q_s heat flux q_h and motile density flux q_s. Above relations in non-dimensional form are:

$$\frac{Nu}{\sqrt{\mathrm{Re}_x}} = -\theta_\eta(0,\tau), \frac{Su}{\sqrt{\mathrm{Re}_x}} = -\varphi_\eta(0,\tau), \frac{Nn}{\sqrt{\mathrm{Re}_x}} = -\psi_\eta(0,\tau). \tag{17}$$

with local Nusselt number (Nu), Sherwood number (Su)and motile density number (Nn).

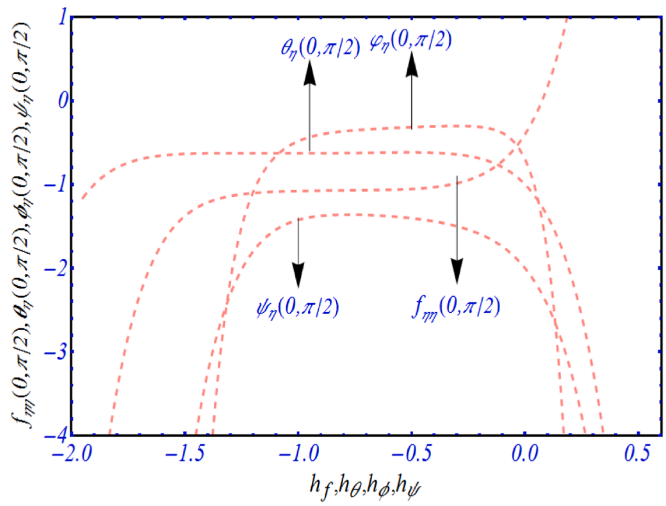

Fig. 2. h − curves for $f_{\eta\eta}, \theta_\eta, \phi_\eta$ and ψ_η.

3. Homotopy analysis method

The clear insight analysis for current work is possible after simulating the solution of developed dimensionless flow equations. The equations of dimensionless flow problem still partial differential equations which are highly nonlinear in nature also. On this end, the numerical solution is quite complicated and cannot be calculated here. Therefore, we employ the most convergent technique namely homotopy analysis method (HAM) to propose the analytical solution. After pioneer work of Liao [43], some novel studies for which solution technique is based on HAM algorithm available in the literature [44–47]. The simulation procedure is initiated by imposing following initial approximations:

$$f_0(\eta,\tau) = \sin\tau(1-e^{-\eta}), \ \theta_0(\eta) = e^{-\eta}, \ \varphi_0(\eta) = e^{-\eta}, \chi_0(\eta) = e^{-\eta}. \tag{18}$$

with auxiliary linear operators

$$\pounds_f = \frac{\partial^3}{\partial\eta^3} - \frac{\partial}{\partial\eta}, \ \pounds_\theta = \frac{\partial^2}{\partial\eta^2} - 1, \ \pounds_\varphi = \frac{\partial^2}{\partial\eta^2} - 1, \pounds_\chi = \frac{\partial^2}{\partial\eta^2} - 1, \tag{19}$$

satisfying

$$\pounds_f\left(\sum_{j=0}^{2} A_{j+1}e^{(j-1)\eta}\right) = 0, \tag{20}$$

$$\pounds_\theta\left(\sum_{j=3}^{4} A_{j+1}e^{(-1)^j\eta}\right) = 0, \tag{21}$$

$$\pounds_\varphi\left(\sum_{j=5}^{6} A_{j+1}e^{(-1)^j\eta}\right) = 0. \tag{22}$$

$$\pounds_\phi\left(\sum_{j=7}^{8} A_{j+1}e^{(-1)^j\eta}\right) = 0. \tag{23}$$

where $A_j (j = 1, 2, ..., 9)$ are arbitrary constants.

4. Convergence analysis

A well-established fact about the HAM procedure discloses that the convergence solution is based on the favorable choice of auxiliary parameters values h_f, h_θ h_φ and h_ψ. Therefore, the $h-$curves are prepared in Fig. 2 which aim to show the convergence region. It is notice that for better convergence, the appropriate values con be followed form $-1.6 \le h_f \le -0.2$, $-1.6 \le h_\theta \le 0.0$, $-1.1 \le h_\varphi \le 0$ and $-1 \le h_\psi \le 0$.

Table 1
Comparison of $f_{\eta\eta}(0,\tau)$ when $S = 1Ha = 12$, $\alpha = \lambda = N_\alpha = N_\beta = 0$.

τ	Zheng et al. [40]	Abbas et al. [41]	Present results
$\tau = 1.5\pi$	11.678656	11.678656	11.678658
$\tau = 5.5\pi$	11.678706	11.678707	11.678704
$\tau = 9.5\pi$	11.678656	11.678656	11.678653

5. Results verification

The verification of obtained analytical results is quite necessary before explore the physical significance of modeled problem. Table 1 is prepared to verify the solution accuracy with the results of Zheng et al. [40] and Abbas et al. [41] as a limiting case. A convincing accuracy of obtained results is noticed with these studies.

6. Physical analysis of results

The flow parameters which affect the velocity, temperature, concentration and microorganisms distributions are physically illustrated in this section. In order to sketch the various curves for these fundamentally involve parameters, the assign fixed values are $\alpha = 0.3$, $M = 0.2$, $S = 0.2$, $\lambda = 0.3$, $N_\beta = 0.3$, $Pr = 0.3$, $\delta = 0.4$, $Pe = 0.2$, $K = 0.2$, $Nb = 0.4$, $Nt = 0.3$, $N_\alpha = 0.5$, $E = 0.5$, $Sc = 0.2$, $Lb = 0.3$, $\Omega = 0.3$ and $\tau = \pi/2$.

6.1. Velocity profile

Fig. 3**(a-b)** reports the dynamic of viscoelastic parameter K and buoyancy ratio constant N_α for velocity profile f_η which is varies against time τ. Fig. 3**(a)** reports that velocity profile shows a declining periodic variation with privilege values of K. Such periodic oscillations in velocity are attributed to the oscillatory motion of surface in its own plane. The graphical simulations performed in Fig. 3**(b)** reveal that when N_α is maximum, a decreasing periodical acceleration in velocity has been

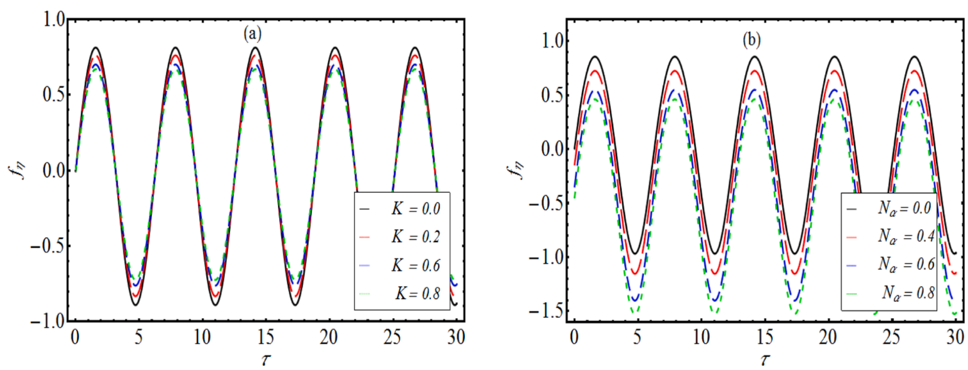

Fig. 3. Velocity variation with time for (a) K (b) N_α.

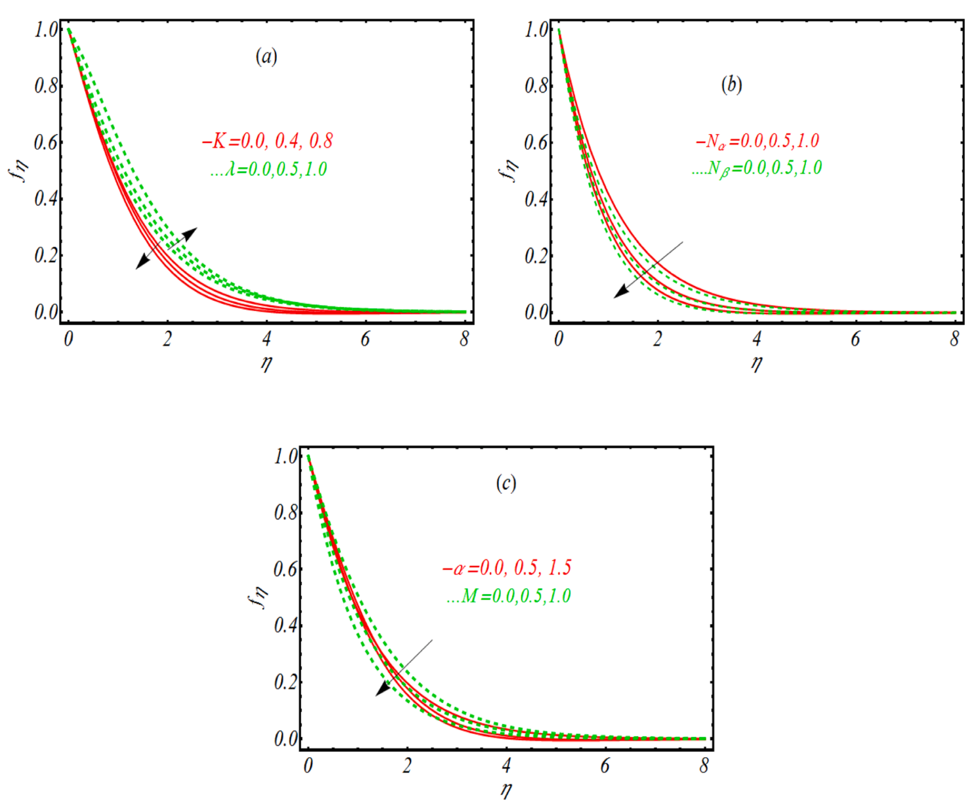

Fig. 4. Velocity profile for (a) K and λ (b) N_α and N_β and (c) α and M.

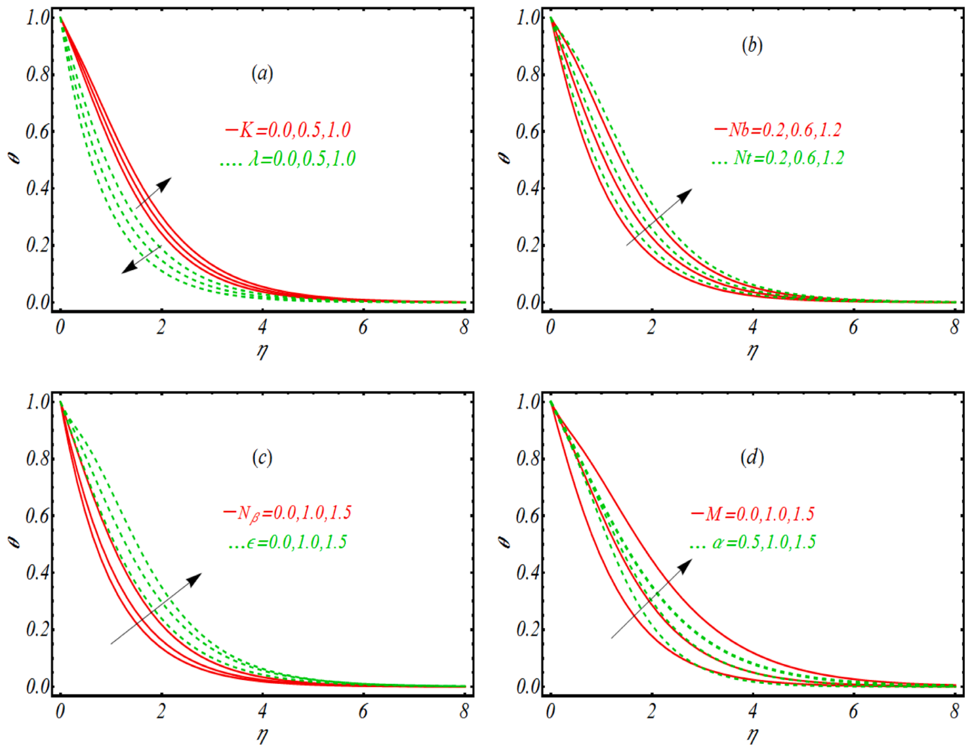

Fig. 5. Temperature profile for (a) K and (b) Nb and Nt (c) N_β and ε, (d) M and α.

resulted. Physically, larger fluctuation in N_α corresponds to more effective buoyancy ratio forces which is resistive to the fluid particles motion. Fig. 4**(a)** aim to visualize the change in f_η for viscoelastic parameter K and mixed convection parameter λ. An improved velocity profile is observed for higher λ. Physically, with increment in K profound to be lower velocity profile due to presence of effective viscous forces. In order to inspect the physical visualization of buoyancy ratio constant N_α and bioconvection Rayleigh number N_β on f_η, Fig. 4**(b)** has been worked out. Since both parameters involve the buoyancy ratio forces which control the movement of fluid particles. Therefore, when maximum value to both flow parameters is intended, the velocity distribution slows down. The graphical illustration of viscosity parameter α and Hartmann number M for f_η is conversed in Fig. 4**(c)**. Since for present flow situations, the viscosity of fluid particles is assumed to be

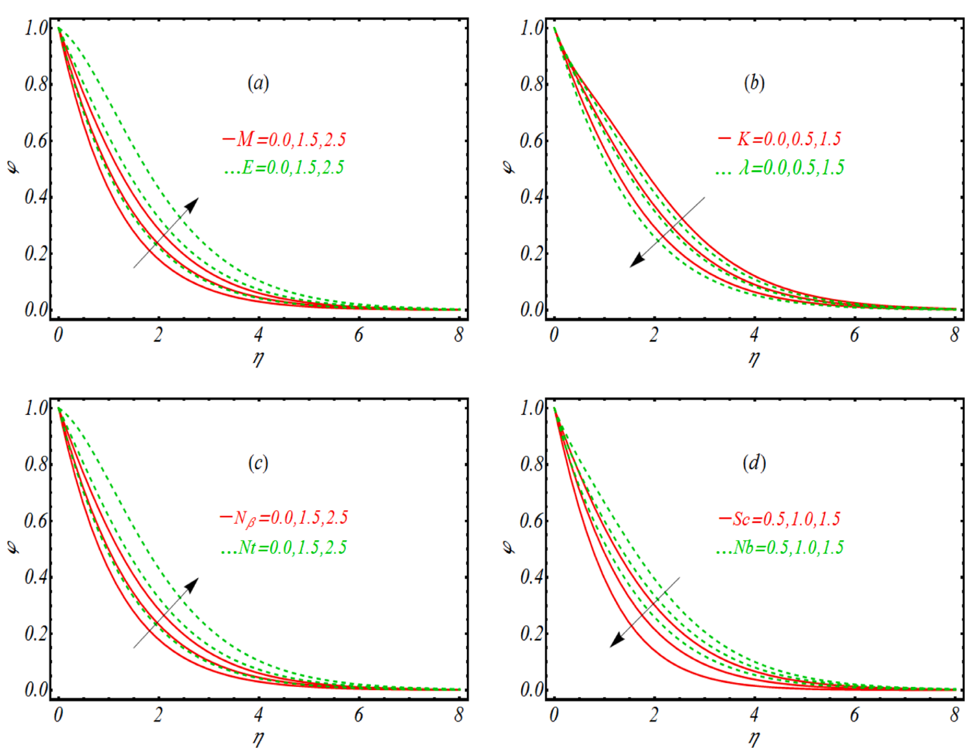

Fig. 6. Concentration profile for (a) M and E, (b) K and λ (c) N_α and N_t (d) Sc and Nb.

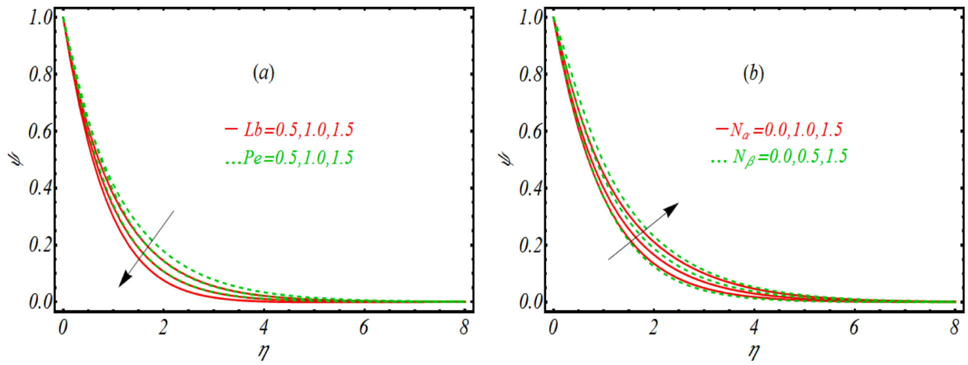

Fig. 7. Microorganisms profile for (a) σ and *Pe*(b) N_α and N_β.

temperature dependent therefore appliances of viscosity parameter is quite necessary. For increasing variation in α, decreasing fluid particles velocity is resulted. Similarly, the impact of Hartmann number *M* is quite similar due to interaction of Lorentz forces which reduce the base liquid velocity.

6.2. Temperature profile

The thermal enhancement based on non-Newtonian nanoparticles is addressed in this investigation with temperature dependent viscosity. Now we execute the change in temperature distribution θ due to influence of viscoelastic parameter *K*, mixed convection constant λ, Brownian motion constant *Nb*, thermophoresis parameter *Nt*, bioconvection Rayleigh number N_β, variable thermal conductivity ε, Hartmann number *M* and viscosity parameter α. Fig. 5**(a)** examine the substantial change in θ due to *K* and λ. The utilization of viscoelastic parameter *K* results a progressive nanoparticles temperature due to viscous forces involvement. However, this change in θ is efficiently control with mixed convection parameter λ. Fig. 5**(b)** claims in improved nanoparticles temperature due to two fundamental nanofluid parameters *Nb* and *Nt*. In the aforementioned illustration, Fig. 5b, temperature distribution is shown to be an increasing property of *Nb* and *Nt*. The Brownian motion constant and thermophoretic parameter play a significant contributions for examining the thermal performances of metallic nano-particles. The thermo-migration feature of nanoparticles is quite essential in various fluid which a details literature is available in refs. [48,49]. Fig. 5**(c)** justifies the dynamic of bioconvection Rayleigh number N_β and variable thermal conductivity parameter ε on θ. Both parameters enable to raise the nanoparticles heat transportation process. Therefore, it is concluded that, consideration of variable thermal conductivity is more effective to improve the nanoparticles temperature. The raise in θ due to N_β is justified with buoyancy forces effects. The joint physical consequences of Hartmann number *M* and viscosity parameter α on θ are visualized in Fig. 5**(c).** An increment in θ is noted for both *M* and α is justified from this curve. Physically, increasing numerical values of Hartman number *M* depressed the Lorentz force features which reduce the velocity. Due to this deceasing procedure, the fluid particles collide with each other and subsequently the fluid temperature gets large. Moreover, it is also claimed that utilization of temperature dependent viscosity is also better to improve the nanoparticles temperature.

6.3. Concentration profile

Fig. 6**(a)** specified the altered concentration distribution ϕ for Hartmann number *M* and activation energy *E*. The concentration of nanofluid is increasing function of both flow parameters even more progressive for activation energy parameter *E*. Physically, the activation energy reports the reaction initiation for many species. The activation energy increases the concentration process. The declining profile of ϕ is testified for viscoelastic parameter *K* and mixed convection constant λ and are reported in Fig. 6**(b)**. The graphical observation predicted in Fig. 6**(c)** reveal the attention of buoyancy ratio constant N_α and thermophoresis constant N_t. The concentration increases with N_α which physically defended because of buoyancy forces. Similarly, the thermophoretic phenomenon also gets down the improved nanofluid concentration. Fig. 6**(d)** depressed the Schmidt number *Sc* and Brownian motion parameter *Nb* impact on ϕ. To elaborate the physical aspects of such declining trend due to *Sc*, it is justified fact that mass diffusivity is minimum when *Sc* attains leading variation.

6.4. Microorganisms profile

The novel Peclet number *Pe* and biconvection Lewis number *Lb* on microorganisms distribution ψ has been depicted in Fig. 7**(a)**. The effeteness variation of both *Pe* and *Lb* reduces ψ. Since because of higher *Pe* variation, the micro-organisms diffusivity get minimum which reduce the profile χ. From Fig. 7**(b)**, the microorganisms distribution ψ enhances with variation of N_α and N_β.

6.5. Physical quantities

The physical aspects of local Nusselt number $-\theta_\eta(0,\tau)$, local Sherwood number $-\phi_\eta(0,\tau)$ and motile density number $-\psi_\eta(0,\tau)$ are numerical studies for various flow constraint parameters in Table 2. An increasing numerical variation parameter in such quantities are notices for mixed convection parameter λ, , viscoelastic parameter *K*, viscosity parameter α and Prandtl number Pr while opposite numerical values are achieved for magneto-porosity parameter *M*, buoyancy ratio constant *Nr* and thermophoresis parameter *Nt*.

Table 2
Illustration of $-\theta_\eta(0,\tau)$, $-\phi_\eta(0,\tau)$ and $-\psi_\eta(0,\tau)$ for different flow parameters.

K	N_α	N_β	α	M	λ	$-\theta_\eta(0,\tau)$	$-\phi_\eta(0,\tau)$	$-\psi_\eta(0,\tau)$
0.20.40.6	0.3	0.1	0.3	0.5	0.5	0.456390.432080.41438	0.423550.413270.40768	0.578660.555540.53154
0.1	0.20.40.6					0.505230.477680.45455	0.477640.445420.41457	0.556570.520980.50896
		0.20.40.6				0.512140.483650.45456	0.444580.428980.40624	0.548110.536320.51614
			0.20.40.6			0.502560.472480.42695	0.463840.436570.41468	0.572130.541120.525333
				0.20.40.6		0.526590.476470.44892	0.441280.4265890.39321	0.555260.534560.505254
					0.20.40.6	0.516530.548950.57035	0.474860.515590.53236	0.555460.5654780.59598

7. Summary

In current work, the bioconvection flow of Walter's B nano-material has been worked out in presence of activation energy. The interest for examining the thermal features of Walter's B nano-materials has inspiring applications like ground pollutions with help of chemicals which are of non-Newtonian nature. Moreover, in order to improve the thermal transportation process, the temperature dependent viscosity and variable thermal consequences are utilized. The convergent technique is successfully employed to execute the solution of this flow model. The physical attributes for fundamental flow parameters is highlighted via various curves. The summarized observation are given below:

- v The buoyancy forces and magnetic force reduce the fluid particles motion in the whole flow domain.
- v The velocity also reduces with higher impact of viscosity parameter.
- v The consideration of temperature dependent viscosity and variable thermal conductivities are more effective for enhancement of nanoparticles temperature.
- v The concentration distribution get slow variation for Schmidt number and Brownian motion constant.
- v The microorganism for nanofluid increases for buoyancy ratio constant and bioconvection Rayleigh number.
- v The fundamental results for current contribution may utilize applications in enhancement of thermal extrusion processes, biofuels applications, enzyme processes, bio-technology, etc.

Declaration of Competing Interest

The authors declared that they have no conflict of interest and the paper presents their own work which does not been infringe any third-party rights, especially authorship of any part of the article is an original contribution, not published before and not being under consideration for publication elsewhere.

Acknowledgment

The authors extend their appreciation to the deanship of scientific research at King Khalid University for funding this work through research groups program under grant number (R.G.P. 2/77/41).

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