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Numerical simulation of AA7072-AA7075/water-based hybrid nanofluid flow over a curved stretching sheet with Newtonian heating: A non-Fourier heat flux model approach



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ABSTRACT

Hybrid nanofluids are widely used in various engineering, manufacturing as well as bio-medical fields. Inspired from these applications, we discussed the flow of AA7072-AA7075/water-based hybrid nanofluid over a curved stretching sheet using non-Fourier heat flux model. Additionally, heat transference is analysed for two different boundary conditions namely, Newtonian heating (NH) and constant wall temperature (CWT). The governing partial differential equations (PDEs) are reduced into ordinary differential equations (ODEs) by opting appropriate similarity transformations. Then they are numerically solved by using Runge-Kutta-Fehlberg's fourth fifth order (RKF-45) technique by adopting shooting method. Behaviour of various parameters on flow and thermal gradients are deliberated through graphs. Also, skin friction and Nusselt number are examined graphically. Results reveal that, velocity increases for higher values of curvature parameter. Thermal distribution of assumed liquid is more in Newtonian heating case when compared to common wall temperature case for diverse values of curvature parameter and thermal relaxation parameter. The upsurge in values of curvature parameter, Newtonian heating parameter and thermal relaxation parameter improves the rate of heat transfer.

Novelty of analysis: With impressed thermal performances, the nanoparticles are assumed as an effective procedure of enhancing the conductivity performances of many normal liquids. On this end, many authors performed various investigations to disclose the thermal aspects of various nanoparticles like single-walled carbon nano-tubes, multi-walled carbon nanotubes (MWCNTs), copper oxide, ferrofluid, silicon dioxide etc. However, the thermal characteristics on AA7072-AA7075/water-based hybrid nanoparticles is not analyzed and presented in the literature. This theoretical analysis access the thermal mechanism of such types of AA7072-AA7075 nanoparticles to improve the heat transfer phenomenon. A comparative analysis for improvement in heat transfer is also presented as compared to the traditional viscous materials.

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1. Introduction

The eminent rate of heat transfer is the major demand from large scale industries and other engineering sectors. This provoked the researchers to introduce a special type of the nanofluid in which dual diverse types of the nanosized particles are suspended in the particular carrier liquid. This category of nanofluid is

* Corresponding author. E-mail address: chuyuming@zjhu.edu.cn (Y.-M. Chu). identified as a hybrid nanofluid. Generally, thermal conductivity of normal nanofluids is lesser when compared to hybrid nanofluids. Hybrid nanofluids are widely utilized in nanotechnological applications. Initially, the nanofluid study was pioneered by choi [1]. Later, several researchers examined the flow of nanofluids over different surfaces. Makinde and Animasaun [2] examined bioconvective stream of fluid with suspended nanoparticles over an upper horizontal surface. Hayat et al. [3] analyzed the convective heat and mass transference in a nanofluid stream instigated by a curved extending sheet. Hybrid nanofluids have excellent Nomenclature

u. v	Velocity components	р	Pressure
R	Distance	Р	Dimensionless pressure
а	Stretching constant	Re	Local Reynolds number
$f'(\eta)$	Dimensionless velocity	Pr	Prandtl number
ϕ_1, ϕ_2	Solid volume fractions of nanoparticles	ρ	Density
(ρC_p)	Specific heat capacity	γ	Newtonian heating parameter
Ď	Diffusion coefficient		
k	Thermal conductivity	Subscrig	ot
r, s	Coordinates	f	Fluid
v	Kinematic viscosity	bf	Base fluid
λ_2	Thermal relaxation parameter	hnf	Hybrid nanofluid
C_f	Skin friction coefficient	∞	Ambient
Т	Temperature	w	Wall/surface
$\theta_1(\eta), \theta_2$	(η) Dimensionless temperature	S_1, S_2	Solid particles
λ_1	Relaxation time of heat flux.	- / -	*
μ	Dynamic viscosity of a liquid		
κ	Curvature parameter		

thermophysical properties which helps in controlling the rate of heat transfer that is compatible with various mechanical and engineering industries. Inspired by these features, numerous researchers examined the hybrid nanofluid flow through diverse surfaces. Das et al. [4] examined the magnetohydrodynamic stream of Cu-Al₂O₃/water hybrid nanoliquid in a porous channel with entropy production. Olatundun and Makinde [5] numerically analysed the Blasius stream of water-based hybrid nanoliquid over a convectively heated surface. Kandasamy et al. [6] evaluated the thermal transfer of water based-AA7075 nanofluid on taking account of electric field. Kumar et al. [7] explored the aspects of magnetic dipole on hybrid nanoliquid stream with suspension of diverse nanoparticles combination through stretched cylinder. Gowda et al. [8] pondered the influence of particle deposition on hybrid nanoliquid flow past an upward/downward moving rotating disk. Tlili et al. [9] scrutinised the nanofluid flow by using AA7072 and AA7075 alloys suspended in methanol as base fluid on taking account of magnetic effect and slip conditions. Jayadevamurthy et al. [10] scrutinized the encouragement of chemical reaction on the liquid stream with the suspension of dual nanoparticles through a moving rotating disk.

The core temperature conduction was initially discussed by Fourier. After some years many scientists begin to study with this concept. By using the relaxation time Cattaneo [11] changed the Fourier rule, which has the paradox of heat conduction as one of the major drawbacks. To overcome the limitation of this, Christov [12] advanced the Fourier model by introducing relaxation time and called it as the Cattaneo -Christov heat flux (CCHF) model. Inspired by the features of this model, several researchers examined the impact of CCHF on diverse liquid streams. Hayat et al. [13] explored the effect of CCHF on viscous liquid stream past a curved stretchy sheet. Irfan et al. [14] deliberated the consequence of CCHF on Carreau liquid stream past a stretchy surface. Hayat et al. [15] explored the impact of non-Fourier heat flux on Jeffrey liquid stream instigated by a curved stretchy sheet. Ali et al. [16] exemplified the impact of CCHF on magnetohydrodynamic stream of Carreau nanoliquid through a stretchy surface. The encouragement of CCHF on stream of Maxwell liquid through a stretchy cylinder was scrutinized by Khan et al. [17]. Christopher et al. [18] deliberated the impact of CCHF on hybrid nanoliquid flow over a stretched surface.

The subject of fluid flow over stretching sheet research often encounters real life problems that have attracted great interest from scientists because of their high value in the fields such as metal extrusion, micro fluidics, transportation, manufacturing, fiberglass production, thermal wrapping, paper production, glass and blasting acoustics. Nowadays, several researchers are interested in scrutinising diverse liquid streams past through curved stretchy sheet. Sajid et al. [19] inspected the viscous liquid stream originated by a curved extending sheet. Hayat et al. [20] scrutinised the Newtonian heating and chemical reaction effects on the Carreau liquid stream through a curved extending sheet. Saif et al. [21] deliberated the liquid stream past a curved extending sheet with porous medium. Nagaraja and Gireesha [22] explored the encouragement of chemical reaction on Casson liquid stream past a curved stretchy sheet. Gireesha et al. [23] explored the effect of heat production on nanoliquid stream past a curved stretchy surface with porous medium.

The heating in which heat transference from surface with determinate amount of heat capacity, that is proportionate to the surface temperature is called as Newtonian heating. It has several practical applications in petroleum industry, conjugate heat transference around fins, solar radiations and designing heat exchangers. Inspired by these applications, numerous researchers deliberated the encouragement of Newtonian heating on diverse liquid streams. The concept of Newtonian heating in fluid flow was initially discussed by Merkin [24]. Makinde [25] deliberated the influence of Newtonian heating on MHD stream of liquid. Hayat et al. [26] deliberated the Newtonian heating effect on second grade liquid stream past a stretchy surface. Das et al. [27] deliberated the Newtonian heating and radiation effects on magnetohydrodynamic stream of nanoliquid past a stretchy sheet. Aleem et al. [28] examined the chemical reaction effects on nanoliguid streams with Newtonian heating. Ahmad and Nadeem [29] examined the influence of Newtonian heating on micropolar hybrid nanoliquid stream. Muhammad et al. [30] inspected the encouragement of Newtonian heating on hybrid nanoliquid stream through a curved surface with viscous dissipation.

Based on the available literature, studies on the steady flow of hybrid nanofluids are very rare and none of the published articles individually discussed the effects of Newtonian heating along with non-Fourier heat flux effects on curved stretching sheet. The liquid suspended with AA7072 – AA7075 alloys as nanoparticles along with water as a base fluid is accounted in this study. The results show the comparison of Newtonian heating and common wall temperature effects on hybrid nanoliquid stream. Hence, to bridge this gap, the current study focuses on the potent variables impact on the fluid characteristics of hybrid nanofluid within the boundary layer.

2. Mathematical formulation

Consider a two-dimensional, incompressible fluid motion of the hybrid nanofluid over a curve shaped stretching sheet coiled in circle with the radius R. The sheet is stretched along a semicircle of radius R by two equal and opposite forces applied along the *s*-direction by keeping the origin fixed and *r*-direction along with the velocity $u = u_w(s) = as$, where *a* is the stretching constant. Further, the flow, and effective thermal conductivity is characterized by the modified Fourier heat flux theory. Additionally, heat transfer analysis is carried out for two different boundary constraints namely, Newtonian heating (NH) and constant wall temperature (CWT). Due to curved stretching sheet the governing equations are modelled by curvilinear coordinates *s* and *r*. Let, *T*_wbe constant temperature of the liquid near the sheet. The temperature of the ambient fluid is T_{∞} . The described flow pattern is shown in Fig. 1.

Under the above assumptions the governing equations for hybrid nanofluids are as follow ([5,15,18;20]):

$$\frac{\partial}{\partial r}\{(r+R)v\} + R\frac{\partial u}{\partial s} = 0,\tag{1}$$

$$\frac{1}{r+R}u^2 = \frac{1}{\rho_{hnf}}\frac{\partial p}{\partial r},\tag{2}$$

$$\nu \frac{\partial u}{\partial r} + \frac{R}{r+R} u \frac{\partial u}{\partial s} + \frac{u\nu}{r+R}$$
$$= -\frac{1}{\rho_{hnf}} \frac{R}{r+R} \frac{\partial p}{\partial s} + \nu_{hnf} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R} \frac{\partial u}{\partial r} - \frac{1}{(r+R)^2} u \right], \tag{3}$$

$$\left. \begin{array}{c} \nu \frac{\partial T}{\partial r} + \frac{R}{r+R} u \frac{\partial T}{\partial s} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R} \frac{\partial T}{\partial r} \right] \\ -\lambda_1 \left(\begin{array}{c} \nu^2 \frac{\partial^2 T}{\partial r^2} + u^2 \left[\frac{R}{r+R} \right]^2 \frac{\partial^2 T}{\partial s^2} + \left\{ \nu \frac{\partial \nu}{\partial r} + \frac{R}{r+R} u \frac{\partial \nu}{\partial s} \right\} \frac{\partial T}{\partial r} + \\ \left\{ \frac{R}{r+R} \nu \frac{\partial u}{\partial r} + u \left[\frac{R}{r+R} \right]^2 \frac{\partial u}{\partial s} \right\} \frac{\partial T}{\partial s} + 2 \frac{R}{r+R} u \nu \frac{\partial^2 T}{\partial r \partial s}. \end{array} \right) \right\}$$
(4)

Related boundary conditions are:

$$u = u_w(s) = as, \ v = 0, \ T = T_w(CWT), \ -\frac{\partial T}{\partial r} = h_s T(NH) \ at \ r$$

= 0, (5)

$$u \to 0, \frac{\partial u}{\partial r} \to 0, \ T \to T_{\infty}, \ as \ r \to \infty.$$
 (6)

The following similarity transformation is applied to governing equations:

$$u = asf'(\eta), \ v = \frac{-R}{r+R} \sqrt{av_f} f(\eta), \ \eta = \sqrt{\frac{a}{v_f}} r,$$

$$\theta(\eta) = \frac{T-T_{\infty}}{T_w - T_{\infty}} (CWT), \ \theta(\eta) = \frac{T-T_{\infty}}{T_{\infty}} (NH),$$

$$p = \rho_f a^2 s^2 P(\eta).$$

$$(7)$$

Here, the density, thermal conductivity, dynamic viscosity and specific heat capacitance of the hybrid nanofluid are given by ([31,32]):

$$\begin{split} \rho_{hnf} &= \left[(1-\phi_1)\rho_f + \phi_1 \rho_{s_1} \right] (1-\phi_2) + \rho_{s_2} \phi_2, \\ k_{hnf} &= \frac{2k_{nf} + k_{s_2} + (k_{s_2} - k_{nf}) 2\phi_2}{2k_{nf} + k_{s_2} - \phi_2 (k_{s_2} - k_{nf})} k_{bf}, k_{bf} \\ &= \frac{k_{s_1} + 2k_f - 2\phi_1 (k_f - k_{s_1})}{k_{s_1} + 2k_f + \phi_1 (k_f - k_{s_1})} k_f, \end{split}$$

$$\mu_{hnf} = \frac{r_{f}}{(1-\phi_{1})^{2.5}(1-\phi_{2})^{2.5}},$$

$$(\rho C_{p})_{hnf} = \left[(1-\phi_{1})(\rho C_{p})_{f} + \phi_{1}(\rho C_{p})_{s_{1}} \right] (1-\phi_{2}) + (\rho C_{p})_{s_{2}}\phi_{2}.$$

After the implementation of similarity variables, Eq. (1) is automatically satisfied and remaining Eqs. (2-4) along with boundary constraints ((5), (6)) are transformed into following forms:

$$\varepsilon_1 P' - \frac{f'^2}{\eta + \kappa} = 0, \tag{8}$$

$$\varepsilon_{1} \frac{2\kappa}{\eta + \kappa} P = \varepsilon_{2} \left[f''' + \frac{1}{\eta + \kappa} f'' - \frac{1}{(\eta + \kappa)^{2}} f' \right] + \frac{\kappa}{\eta + \kappa} f f'' - \frac{\kappa}{\eta + \kappa} (f')^{2} + \frac{\kappa}{(\eta + \kappa)^{2}} f f',$$
(9)

$$\frac{k_{\text{hnf}}}{k_{f}} \varepsilon_{3} \frac{1}{\Pr} \left[\theta'' + \frac{1}{\eta + \kappa} \theta' \right] + \frac{\kappa}{\eta + \kappa} f \theta' - \lambda_{2} \left\{ \left[\frac{\kappa}{\eta + \kappa} \right]^{2} \left(f^{2} \theta'' + f f' \theta' - \frac{1}{\eta + \kappa} f^{2} \theta' \right) \right\} = 0.$$
(10)

Where,

$$\begin{split} & \mathcal{E}_{1} = \frac{1}{(1-\phi_{2})\left[(1-\phi_{1})+\phi_{1}\frac{\rho_{s_{1}}}{\rho_{f}}\right]+\phi_{2}\frac{\rho_{s_{2}}}{\rho_{f}}}, \\ & \mathcal{E}_{2} = \frac{1}{(1-\phi_{1})^{2.5}(1-\phi_{2})^{2.5}\left\{(1-\phi_{2})\left[(1-\phi_{1})+\phi_{1}\frac{\rho_{s_{1}}}{\rho_{f}}\right]+\phi_{2}\frac{\rho_{s_{2}}}{\rho_{f}}\right\}}, \\ & \mathcal{E}_{3} = \frac{1}{(1-\phi_{2})\left[(1-\phi_{1})+\phi_{1}\frac{(\rho C_{p})_{s_{1}}}{(\rho C_{p})_{f}}\right]+\phi_{2}\frac{(\rho C_{p})_{s_{2}}}{(\rho C_{p})_{f}}}. \end{split}$$

The corresponding reduced boundary conditions are as follow:

$$\begin{cases} f'(0) = 1, f(0) = 0, \ \theta(0) = 1(CWT), \\ f'(\infty) \to 0, f''(\infty) \to 0, \ \theta(\infty) \to 0. \end{cases}$$
(11)



Fig. 1. Geometrical representation of flow.

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Along with another case

$$\theta'(\mathbf{0}) = -\gamma(\mathbf{1} + \theta(\mathbf{0}))(NH). \tag{12}$$

Using Eq. (9) pressure can be determined. Further, following equation is obtained after eliminating pressure $P(\eta)$ from Eqs. (8) and (9):

$$\varepsilon_{2} \left[f^{iv} + \frac{2f'''}{\eta + \kappa} - \frac{f''}{(\eta + \kappa)^{2}} + \frac{f'}{(\eta + \kappa)^{3}} \right] + \frac{\kappa}{\eta + \kappa} \left(ff''' - (f'')f' \right) \\ + \frac{\kappa}{(\eta + \kappa)^{2}} \left(ff'' - (f')^{2} \right) - \frac{\kappa}{(\eta + \kappa)^{3}} ff' = 0.$$
(13)

Where,

$$\kappa = \sqrt{\frac{a}{\nu_f}} R, \Pr = \frac{(\rho C_p)_f \nu_f}{k_f}, \lambda_2 = \lambda_1 a, \gamma = h_s \sqrt{\frac{\nu_f}{a}}$$

The engineering quantities like Skin friction and Nusselt number in its dimensionless form are as follow:

$$\sqrt{R_e}C_f = \frac{f''(0) - \frac{f'(0)}{\kappa}}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}},\tag{14}$$

$$\frac{Nu}{\sqrt{\text{Re}}} = \frac{-k_{\text{hnf}}}{k_f} \theta'(\mathbf{0}).$$
(15)

Here, $\text{Re} = \frac{as^2}{v_r}$ Local Reynolds number.

3. Results and discussions

This segment covers the physical explanation of various dimensionless parameters on the described boundary value problem. In this current study, we investigated the impact of Newtonian heating on hybrid nanoliquid stream over a curved extending sheet with modified Fourier heat flux model. Here, we considered aluminum alloys as nanoparticles suspended in carrier fluid water. Further, AA7072 alloy is an amalgamated mix of Zinc and Aluminum in the proportion 1 and 98 respectively, with additional ferrous, metals Silicon and Copper. Similarly, AA7075 is a combination of Copper, Magnesium, Zinc, and Aluminum, in the proportion of, ~1 ~3, ~6, and ~90 correspondingly with added metals Magnesium and Silicon ferrous. The physical characteristics of these alloys and carrier fluid are tabulated in Table 1. The framedPDEs are reduced to ODEs by means of apt similarity variables. The reduced ODEs are numerically solved and analyzed graphically. The rate of heat transportation phenomenon is portrayed in the resulting hybrid nanofluids. The domination of several dimensionless parameters on velocity and thermal gradients are deliberated graphically. The pictorial results of the flow, thermal profile, skin friction and Nusselt number are found to get an obvious insight of the existing boundary flow problem. In this investigation, numerical estimates presented the validation of skin friction and is tabulated in Table 2 with observations reported by Sajid et al.

Table 1

Thermo-physical properties of nanoparticles and water ([3,5]).

Nanoparticles	AA7072	AA7075	Water
$\rho(kg/m^3)$	2720	2810	997.1
$c_p(J/kgK)$	893	960	4179
k(W/mk)	222	173	0.613

Table 2

Validation of the Numerical Scheme. Numerical values of $-\sqrt{R_e}C_f$ when $\phi_1 = \phi_2 = 0$.

κ	Sajid et al. [19]	Present Work
5	0.75763	0.754505
10	0.87349	0.872445













Fig. 4. Influence of λ_2 over $\theta(\eta)$.

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Fig. 5. Influence of Pr over $\theta(\eta)$.

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Fig. 6. Influence of ϕ_2 over $\theta(\eta)$.

[19]. A superb agreement among the results is found for the extraordinary instance of the present problem.

The fluctuation in velocity profile for varied values of κ is illustrated in Fig. 2. Here, improvement in κ progresses the velocity of the fluid motion. Physically, the inclination in κ reasons for the bigger radius and liquid travels quicker across the sheet which automatically upsurges the velocity gradient. Fig. 3 reveals the impact of κ on thermal gradient for two different temperature cases. One can notice from plotted figure that, inclination in curvature parameter declines the thermal gradient. Further, rate of declination in thermal gradient is slower for common wall temperature case when compared to Newtonian heating case. The sway of λ_2 on thermal gradient for common wall temperature and Newtonian heating cases is portrayed in Fig. 4. Plotted figure shows that, inclined values of λ_2 decline the thermal gradient. Physically, for inclination in λ_2 displays a non-conducting behavior which is accountable for decay in thermal gradient. Moreover, rate of declination in thermal gradient.

nation in thermal gradient is faster for NH case when compared to CWT case.

Variation in thermal profile against Prandtl number for both cases is portrayed in Fig. 5. The enhanced *Pr* reduces the fluid temperature. Physically, the higher *Pr* numbers possess lower thermal conductivity which results in decay of thermal gradient. On the other hand, lower *Pr* numbers have high conductivity of rising temperature, which upsurges the temperature of the boundary layer flow. Moreover, rate of declination in thermal gradient of assumed hybrid nanofluid is faster for NH case when compared to CWT case. The impact of ϕ_2 on thermal gradient for common wall temperature and Newtonian heating cases is represented in Fig. 6. One can detect from plotted graph that, upsurge in solid volume fraction ϕ_2 increases the thickness of the boundary layer and improves the heat transference. Further, rate of inclination in heat transfer of assumed flow is faster for common wall temperature case when compared to Newtonian heating case.



Fig. 7. Change in C_f over ϕ_2 for various values of κ .

2.02 2.04 2.06 2.08

2.10 2.12 2.02 2.04

2.06 2.08 2.10

2.12

2.02 2.04

2.06 2.08

2.10 2.12

2.02 2.04

2.06

2.08

2.10



Fig. 8. Change in *Nu* over λ_2 for various values of κ .



Fig. 9. Change in *Nu* over γ for various values of ϕ_2 .

The impact of several dimensionless parameters on skin friction and Nusselt number are deliberated with the help of threedimensional plots (see Figs. 7–9). The influence of ϕ_2 on friction factor versus κ is demonstrated in Fig. 7. Here, upsurge in ϕ_2 improves the surface drag force. Moreover, skin friction acts as a growing function of κ . The variation in Nusselt number versus κ for diverse values of λ_2 is illustrated in Fig. 8. Plotted figure concludes that, upsurge in λ_2 improves the rate of heat transfer. Further, Nusselt number acts as a growing function of κ . The fluctuation in*Nu* over γ for varied values of ϕ_2 is illustrated in Fig. 9, Plotted figure signifies that, rise in values of γ improves the rate of heat transference. Moreover, Nusselt number acts as an increasing function of $\phi_{\rm 2}.$

4. Final remarks

In this study, we investigated the influence of modified Fourier heat flux on hybrid nanofluid stream over a curved stretching sheet with Newtonian heating. Here, we considered aluminium alloys AA7072 and AA7075 suspended in water as carrier fluid. The behaviour of velocity, and effective thermal gradients are analysed graphically. Also, skin friction and rate of heat transference are deliberated by using suitable three-dimensional graphs. The associated outcomes are distinguished from this inspection are as follow:

- 1. Escalating values of curvature parameter increases the velocity gradient.
- 2. Thermal distribution of assumed flow for Newtonian heating case is more when compared to common wall temperature case for increase in values of curvature parameter and thermal relaxation parameter.
- 3. Thermal distribution upsurges with upsurge in volume fraction.
- 4. The enhanced Prandtl number values decay the fluid temperature and rate of declination in thermal gradient of assumed hybrid nanofluid is faster for NH case when compared to CWT case.
- 5. The upsurge in curvature parameter, Newtonian heating parameter and thermal relaxation parameter improves the rate of heat transfer.

CRediT authorship contribution statement

B.C. Prasannakumara: Conceptualization, Data curation, Project administration, Supervision, Visualization, Writing - review & editing. **Yu-MingChu:** Conceptualization, Resources, Supervision, Visualization, Writing - review & editing. **M Ijaz Khan:** Data curation, Resources. **R.J. Punith Gowda:** Formal analysis, Funding acquisition, Investigation, Software, Writing - original draft. **G.K. Ramesh:** Formal analysis, Methodology, Validation. **R. Naveen Kumar:** Formal analysis, Validation, Writing - original draft. **J.K. Madhukesh:** Funding acquisition, Investigation, Methodology, Software. **Sami Ullah khan:** Software, Writing - original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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