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FUZZY SYSTEMS AND THEIR MATHEMATICS

Applications of improved spherical fuzzy Dombi aggregation operators in decision support system

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Abstract
Spherical fuzzy sets are an extension of various fuzzy concepts and demonstrate fuzzy opinion using membership, abstinence, nonmembership and refusal degrees with relaxed conditions, and it is a better mathematical tool to deal with uncertain and vague information. Recently, Dombi operational laws for spherical fuzzy numbers (SPFNs) are developed for multi-attribute decision-making purpose. In this article, some limitations of the said Dombi operational laws for SPFNs are investigated as the aggregated information that came out using existing aggregation operators deviate from the range. Therefore, in this paper, we aim to present some improved Dombi operational laws for SPFNs. Further, keeping the advantages of the power aggregation operators that it takes into account the relationship of the information being aggregated, we aim to develop the spherical fuzzy Dombi power average operator, the spherical fuzzy Dombi weighted power average operator, spherical fuzzy Dombi power geometric operator, and spherical fuzzy Dombi weighted power geometric operator and their desirable properties are discussed. The main advantage of these developed Dombi power aggregation operators is that they eliminate the effect of awkward data and are more flexible due to general parameters involved in aggregation process. Moreover, based on these Dombi power aggregation operators, a novel multi-attribute group decision-making approach is instituted followed by a numerical example to show the practicality and effectiveness of the proposed approach and comparison with the existing approaches is also given.

Keywords Spherical fuzzy set · Power aggregation · Dombi t-norm · Dombi t-conorm · MAGDM

Introduction

One of the complicatedness of realistic multiple-attribute group decision-making (MAGDM) problems is how to point out the attribute values in fuzzy and indistinct decision-making environments. The theory of fuzzy set (FS) was initiated by Zadeh (1965) as a tool for describing and communicating volatility and fuzziness. Since its inception, FS has attained outstanding attention from scholars all over the world, who premeditated its actual and theoretical aspects. Some recent work on the theory and applications of FSs can be found in differential equations (Arqub et al. 2016, 2017; Arqub and Al-Smadi 2020; Arqub 2017), genetic algorithms (Momani et al. 2016) and supply chain management (Wang et al. 2020; Xiao et al. 2020), etc. After the notion of FS is introduced, a variety of expansions of FSs have been anticipated, such as interval-valued FS (IVFS) (Turksen 1986), which explicated the membership degree on a closed interval in the interval $[0, 1]$, and intuitionistic fuzzy set (IFS) (Atanassov 1986) developed by Atanassov, which explicated the membership degree (MD) and nonmembership degree (NMD) and the sum of these two degrees must be less or equal to one. Therefore, IFS explicates fuzziness and unpredictability

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more extensively than FS. However, the attractive situation originated when the MD and NMD of an object are acceptable from the unit interval, but the sum of these two degrees goes beyond one. In such circumstances, the conventional IFS fails to deal with such information. Therefore, an extensive mathematical model is needed to deal with such types of situations. To deal with the above-defined situation, Yager (2013, 2014) and Yager and Abbasov (2013) developed the idea of Pythagorean fuzzy sets (PyFSs), which could be judged as an extension of IFSs. The only difference between PyFS and IFS is that the sum of squares of the MD and NMD in PyFS is less or equal to one, while the sum of MD and NMD in IFS is less or equal to one. After the initiation of PyFS, various studies have been conducted by many researchers, such as distance measure (Wei and Wei 2018; Peng and Dai 2017; Chen 2018; Zhang 2016), correlation coefficient (Garg 2016a). Zhang and Xu (2014) delivered the comprehensive mathematical expression for PyFS and proposed the idea of PyFN, and then they also proposed a MADM approach based on Pythagorean fuzzy TOPSIS (a technique for order preference by similarity to ideal solution) to deal with PyFNs. Ren et al. (2016) developed the TODIM approach to obtain the optimal alternative in decision-making utilizing PyFNs.

IFS and PyFS have been concerned in different areas. However, there exist some practical cases where the IFS and PyFS are unsuitable to deal with such a situation. For example, there is voting for the selection of vice president among the deans of three faculties, i.e., faculty of basic and applied sciences (FBAS), faculty of engineering and technology (FET), and faculty of management sciences (FMS). For voting, 100 professors are selected from these faculties. Thirty professors vote in favor of FBAS, forty professors vote against FBAS, twenty votes of professors have abstained, and ten professors refuse to vote. In such type of information, IFS and PyFS fail to deal with it. So, to deal with such information, another generalization of IFS was initiated by Cuong and Kreinovich (2013) and named it to picture fuzzy set (PFS). However, in some situation when the positive MD, neutral MD and NMD of an object is acceptable from the unit interval, but the sum of these three degrees goes ahead of one. In such circumstances, the conventional PFS fails to deal with such information. Therefore, a more comprehensive mathematical tool is required to deal with such information. For this purpose, Mahmood et al. (2018) and Ashraf et al. (2019a) developed the concept of spherical fuzzy set (SPFS) which is further an extension of PyFS and PFS. The structure of SPFS is the same as that of PFS but the only change is that the sum of squares of these three degrees must be less or equal to one. So SPFS is a more powerful tool to deal with vague, inconsistent, and uncertain information.

The aggregation operators (AOs) play an important role in decision making. From the last few years, information AOs have attained much more awareness from the scholars, and they initiated different AOs and their extensions, for example, the usual aggregation operator introduced by Xu and Xu and Yager (2007, 2006) can have the capacity to aggregate a set of real numbers into a single real number. Nowadays these usual AOs were further extended by several scholars; for example, Garg (2016b, 2017) initiated generalized PyFWA operator, generalized Pythagorean fuzzy ordered weighted operators, generalized PyFWG operator, a generalized Pythagorean fuzzy ordered weighted geometric operator based on Einstein t-norm and t-conorm and applied these AOs to MADM. Wei (2017) proposed some Pythagorean fuzzy interaction aggregation and applied these to MADM. Zeng et al. (2016) proposed some Pythagorean fuzzy ordered weighted averaging weighted average distance operators and applied these to MCDM. Different AOs have different characteristics. Some AOs considered the interrelationship among input arguments, such as Bonferroni mean (BM) (Bonferroni 1950), Heronian mean (HM) (Sykora 2009), Muirhead mean operators (Muirhead 1902), Maclaurin symmetric mean operator (Maclaurin 1729). These AOs were further extended to deal with various fuzzy environments (Liu and You 2017; Qin and Liu 2014; Li et al. 2016; Xu and Yager 2011; Zhou and He 2012). Some AOs can reduce the influence of awful data such as PA operator initially represented by Yager (2001), and it was extended by many researchers from all over the world to deal with various environments, such as Xu (2011) proposed intuitionistic fuzzy power aggregation operator and applied these to MA group DM (MGDM). Wei and Lu (2018) developed Pythagorean fuzzy power AOs and applied these to MADM. Gündoğdu and Kahraman (2019a, b, c) and Gündoğdu et al. (2019) proposed several methods for SPFS and give its applications in decision making. Jin et al. (2019) developed some spherical logarithmic aggregation operators and give their application in decision support systems.

For aggregating SPFNs, mostly AOs are proposed using algebraic T-norm (TN) and T-conorm (TCN). Recently, Ashraf et al. (2019b) developed Dombi operational laws for SPFNs based on Dombi (1982) TN and TCN and proposed some spherical fuzzy Dombi aggregation operators and applied these to MADM. Generally, Archimedean TN and TCN are the generalizations of Dombi TN, Dombi TCN, and of some other TN and TCN such as algebraic, Einstein, Hamacher, Frank, etc. Dombi TN and TCN have the superiority of general TN and TCN by a general parameter, and this can make the aggregation process more flexible. In recent years, some researchers developed Dombi operational laws for various fuzzy sets and proposed a variety of AOs on these Dombi operational laws and some other aggregation operators in Liu et al. 2018, Chen and Ye 2017, He 2018, Khan et al. 2018, Li et al. 2018, Wu et al. 2018, Zhang et al. 2018, Jana et al. 2018 and Ashraf et al. 2019c). However, the existing Dombi operational laws for SPFNs and the Dombi weighted AOs have several limitations, which will be discussed in Sects. 3 and 4.

Therefore, the main goal of this article is to propose improved Dombi operational laws, improve the existing Dombi AOs and develop some new AOs, such as spherical fuzzy Dombi power averaging operator and its weighted form, spherical fuzzy Dombi power geometric operator, and its weighted form, and discussed several basic properties of these newly proposed AOs and apply them to MAGDM.

To do so, the rest of the article is organized as follows: In Sect. 2, some basic definitions about SPFS, PA operator, Dombi TN, and Dombi TCN are given. In Sect. 3, some limitations in the Dombi operational laws for SPFNs are discussed and some improved operational laws are proposed to over these limitations. In Sect. 4, limitations in existing Dombi AOs are discussed and some improved AOs are proposed. In Sect. 5, two newly spherical fuzzy Dombi power AOs are proposed and discussed for their desirable properties. In Sect. 6, based on these newly developed AOs, a novel approach to MAGDM is developed. In Sect. 7, the conclusion, future work, and references are given.

Preliminaries

In this section, some basic definitions about SPFSs, SPFN, PA operator, Dombi t-norm, and Dombi t-conorm and their related properties are discussed.

Spherical fuzzy sets and their operational laws

In this subsection, definitions of TN and TCN adapted from Klement and Mesiar (2005) and Nguyen and Walker (2018), SPFSs (Mahmood et al. 2018; Ashraf et al. 2019a), SPFNs (Mahmood et al. 2018; Ashraf et al. 2019a) are given.

Definition 1 (Klement and Mesiar 2005; Nguyen and Walker 2018) A function $\overline{\overline{T}}:\Im\times\Im\rightarrow\Im$ is said to be triangular norm (TN) if, for every element, $\overline{\overline{T}}$ satisfies the following axioms:

(1) $\overline{\overline{T}}$ is commutative, monotonic, and associative.

(2) $\overline{\overline{T}}\left(\overline{\overline{t}},1\right)=\overline{\overline{t}}$, for each $\overline{\overline{t}}\in\overline{\overline{T}}$.

where $\Im=[0,1]$ is the unit interval.

Definition 2 (Klement and Mesiar 2005; Nguyen and Walker 2018) A function $\overline{\overline{S}}:\Im\times\Im\rightarrow\Im$ is said to be triangular conorm (TCN) if, for every element, $\overline{\overline{S}}$ satisfies the following axioms:

(1) $\overline{\overline{S}}$ is commutative, monotonic, and associative.

(2) $\overline{\overline{S}}\left(\overline{\overline{s}},0\right)=\overline{\overline{s}}$, for each $\overline{\overline{s}}\in\overline{\overline{S}}$.

where $\Im=[0,1]$ is the unit interval.

In Tables 1 and 2, different types of TN and TCN with their generators are given.

Definition 3 (Mahmood et al. 2018; Ashraf et al. 2019a) Let $\overline{\overline{U}}$ be the universe of discourse set, A SPFS $\overline{\overline{SP}}$ in $\overline{\overline{U}}$ is an object of the form

$$\overline{\overline{SP}}=\left\{\left\langle\overline{\overline{po}}(\overline{\overline{u}}),\overline{\overline{ne}}(\overline{\overline{u}}),\overline{\overline{fl}}(\overline{\overline{u}})\right\rangle|\overline{\overline{u}}\in\overline{\overline{U}}\right\};\tag{1}$$

where $\overline{\overline{po}}(\overline{\overline{u}}):\overline{\overline{U}}\rightarrow[0,1],\overline{\overline{ne}}:\overline{\overline{U}}\rightarrow[0,1]$ and $\overline{\overline{fl}}:\overline{\overline{U}}\rightarrow[0,1]$ are three functions, respectively, representing positive-membership, neutral-membership, and negative-membership degrees of each element $\overline{\overline{u}}\in\overline{\overline{U}}$. The above three functions must satisfy the condition that

$$0\leq\left(\overline{\overline{po}}(\overline{\overline{u}})\right)^2+\left(\overline{\overline{ne}}(\overline{\overline{u}})\right)^2+\left(\overline{\overline{fl}}(\overline{\overline{u}})\right)^2\leq1.$$

The refusal degree of the $\overline{\overline{SP}}$ is denoted by

$$\overline{\overline{\pi}}_{\overline{\overline{SP}}}(\overline{\overline{u}})=\sqrt{1-\left(\overline{\overline{po}}(\overline{\overline{u}})\right)^2+\left(\overline{\overline{ne}}(\overline{\overline{u}})\right)^2+\left(\overline{\overline{fl}}(\overline{\overline{u}})\right)^2}.$$

The triple component $\overline{\overline{sp}}=\left\langle\overline{\overline{po}}(\overline{\overline{u}}),\overline{\overline{ne}}(\overline{\overline{u}}),\overline{\overline{fl}}(\overline{\overline{u}})\right\rangle$ is said to be an SPFN.

A SPFS can be very useful in situations where human opinion is involved as a human opinion can have many forms such as yes, no, abstain and refusal degree. Further, SPFS provide a better range for assigning the membership grades compare to other fuzzy frameworks. Some basic properties of SPFN are illustrated as follows:

Definition 4 (Ashraf et al. 2019a) Let $\overline{\overline{sp}}=\left\langle\overline{\overline{po}},\overline{\overline{ne}},\overline{\overline{fl}}\right\rangle,\overline{\overline{sp}}_1=\left\langle\overline{\overline{po}}_1,\overline{\overline{ne}}_1,\overline{\overline{fl}}_1\right\rangle$ and $\overline{\overline{sp}}_2=\left\langle\overline{\overline{po}}_2,\overline{\overline{ne}}_2,\overline{\overline{fl}}_2\right\rangle$ be any three SPFNs, and $\zeta>0$. Then,

(1) $$\overline{\overline{sp}}_1\oplus\overline{\overline{sp}}_2=\left\langle\sqrt{\left(\overline{\overline{po}}_1\right)^2+\left(\overline{\overline{po}}_2\right)^2-\left(\overline{\overline{po}}_1\right)^2\left(\overline{\overline{po}}_2\right)^2},\overline{\overline{ne}}_1\overline{\overline{ne}}_2,\overline{\overline{fl}}_1\overline{\overline{fl}}_2\right\rangle;\tag{2}$$

Table 1 Different types of TN with their generator

Name of TN	TN	Additive generators
Algebraic	$\overline{\overline{T}}_A\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\overline{\overline{t}}\,\overline{\overline{b}}$	$\overline{\overline{l}}(\overline{\overline{t}})=-\log\overline{\overline{t}}$
Einstein	$\overline{\overline{T}}_E\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\frac{\overline{\overline{t}}\,\overline{\overline{b}}}{1+\left(1-\overline{\overline{t}}\right)\left(1-\overline{\overline{b}}\right)}$	$\overline{\overline{l}}(\overline{\overline{t}})=\log\frac{2-\overline{\overline{t}}}{\overline{\overline{t}}}$
Hamacher	$\overline{\overline{T}}_E\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\frac{\overline{\overline{t}}\,\overline{\overline{b}}}{\wp+(1-\wp)\left(\overline{\overline{t}}+\overline{\overline{b}}-\overline{\overline{t}}\,\overline{\overline{b}}\right)};\wp>0.$	$\overline{\overline{l}}\left(\overline{\overline{t}}\right)=\log\frac{\wp+(1-\wp)\overline{\overline{t}}}{\overline{\overline{t}}};\wp>0.$
Frank	$\overline{\overline{T}}_F\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\log_{\wp}\left(1+\frac{\left(\wp^{\overline{\overline{t}}}-1\right)\left(\wp^{\overline{\overline{b}}}-1\right)}{\wp-1}\right)$	$\wp=1,\overline{\overline{l}}(\overline{\overline{t}})=-\log\overline{\overline{t}}$ $\wp\neq1,\overline{\overline{l}}(\overline{\overline{t}})=-\log\frac{\wp-1}{\wp^{\overline{\overline{t}}}-1}$

Table 2 Different types of TCN with their generator

Name of TCN	TCN	Additive generators
Algebraic	$\overline{\overline{S}}_A\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\overline{\overline{t}}+\overline{\overline{b}}-\overline{\overline{t}}\,\overline{\overline{b}}$	$\overline{\overline{s}}(\overline{\overline{t}})=-\log(1-\overline{\overline{t}})$
Einstein	$\overline{\overline{S}}_E\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\frac{\overline{\overline{t}}+\overline{\overline{b}}}{1+\overline{\overline{t}}\,\overline{\overline{b}}}$	$\overline{\overline{s}}(\overline{\overline{t}})=\log\frac{1+\overline{\overline{t}}}{1-\overline{\overline{t}}}$
Hamacher	$\overline{\overline{S}}_E\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\frac{\overline{\overline{t}}+\overline{\overline{b}}-\overline{\overline{t}}\,\overline{\overline{b}}-(1-\wp)\overline{\overline{t}}\,\overline{\overline{b}}}{1-(1-\wp)\overline{\overline{t}}\,\overline{\overline{b}}};\wp>0.$	$\overline{\overline{s}}\left(\overline{\overline{t}}\right)=\log\frac{\wp+(1-\wp)(1-\overline{\overline{t}})}{(1-\overline{\overline{t}})};\wp>0.$
Frank	$\overline{\overline{S}}_F\left(\overline{\overline{t}},\overline{\overline{b}}\right)=\log_{\wp}\left(1+\frac{\left(\wp^{1-\overline{\overline{t}}}-1\right)\left(\wp^{1-\overline{\overline{b}}}-1\right)}{\wp-1}\right)$	$\wp=1,\overline{\overline{s}}(\overline{\overline{t}})=-\log(1-\overline{\overline{t}})$ $\wp\neq1,\overline{\overline{s}}(\overline{\overline{t}})=-\log\frac{\wp-1}{\wp^{1-\overline{\overline{t}}}-1}$

(2) $$\overline{\overline{sp}}_1\otimes\overline{\overline{sp}}_2=\left\langle\overline{\overline{po}}_1\overline{\overline{po}}_2,\sqrt{\left(\overline{\overline{ne}}_1\right)^2+\left(\overline{\overline{ne}}_2\right)^2-\left(\overline{\overline{ne}}_1\right)^2\left(\overline{\overline{ne}}_2\right)^2},\sqrt{\left(\overline{\overline{fl}}_1\right)^2+\left(\overline{\overline{fl}}_2\right)^2-\left(\overline{\overline{fl}}_1\right)^2\left(\overline{\overline{fl}}_2\right)^2}\right\rangle; \tag{3}$$

(3) $$\zeta\overline{\overline{sp}}=\left\langle\sqrt{1-\left(1-\left(\overline{\overline{po}}\right)^2\right)^{\zeta}},\overline{\overline{ne}}^{\zeta},\overline{\overline{fl}}^{\zeta}\right\rangle; \tag{4}$$

(4) $$\overline{\overline{sp}}^{\zeta}=\left\langle\overline{\overline{po}}^{\zeta},\sqrt{1-\left(1-\left(\overline{\overline{ne}}\right)^2\right)^{\zeta}},\sqrt{1-\left(1-\left(\overline{\overline{fl}}\right)^2\right)^{\zeta}}\right\rangle. \tag{5}$$

In order to compare two SPFNs, Chen (2018) proposed the following comparison laws.

Definition 5 (Ashraf et al. 2019a) Let $\overline{\overline{sp}}=\left\langle\overline{\overline{po}},\overline{\overline{ne}},\overline{\overline{fl}}\right\rangle$ be an SPFN. Then, the score and accuracy functions are defined as follows:

$$\overline{\overline{SE}}=\frac{1}{3}\left(2+\overline{\overline{po}}-\overline{\overline{ne}}-\overline{\overline{fl}}\right);\ \overline{\overline{SE}}\in[0,1], \tag{6}$$

$$\overline{\overline{AC}}=\overline{\overline{po}}-\overline{\overline{ne}},\overline{\overline{AC}}\in[0,1]. \tag{7}$$

Definition 6 (Ashraf et al. 2019a) Let $\overline{\overline{sp}}_1=\left\langle\overline{\overline{po}}_1,\overline{\overline{ne}}_1,\overline{\overline{fl}}_1\right\rangle$ and $\overline{\overline{sp}}_2=\left\langle\overline{\overline{po}}_2,\overline{\overline{ne}}_2,\overline{\overline{fl}}_2\right\rangle$ be any two SPFNs, then the comparison rules are defined as follows:

1. If $\overline{\overline{SE}}(\overline{\overline{sp}}_1)>\overline{\overline{SE}}(\overline{\overline{sp}}_2)$, then $\overline{\overline{sp}}_1$ is greater than $\overline{\overline{sp}}_2$ and is denoted as $\overline{\overline{sp}}_1>\overline{\overline{sp}}_2$;
2. If $\overline{\overline{SE}}(\overline{\overline{sp}}_1)=\overline{\overline{SE}}(\overline{\overline{sp}}_2)$, and $\overline{\overline{AC}}(\overline{\overline{sp}}_1)>\overline{\overline{AC}}(\overline{\overline{sp}}_2)$, then $\overline{\overline{sp}}_1$ is greater than $\overline{\overline{sp}}_2$ and is denoted as $\overline{\overline{sp}}_1>\overline{\overline{sp}}_2$;
3. If $\overline{\overline{SE}}(\overline{\overline{sp}}_1)=\overline{\overline{SE}}(\overline{\overline{sp}}_2)$, and $\overline{\overline{AC}}(\overline{\overline{sp}}_1)=\overline{\overline{AC}}(\overline{\overline{sp}}_2)$, then $\overline{\overline{sp}}_1$ is equal to $\overline{\overline{sp}}_2$ and is denoted as $\overline{\overline{sp}}_1=\overline{\overline{sp}}_2$.

Definition 7 (Ashraf et al. 2019c) Let $\overline{\overline{sp}}_1=\left\langle\overline{\overline{po}}_1,\overline{\overline{ne}}_1,\overline{\overline{fl}}_1\right\rangle$ and $\overline{\overline{sp}}_2=\left\langle\overline{\overline{po}}_2,\overline{\overline{ne}}_2,\overline{\overline{fl}}_2\right\rangle$ be any two SPFNs, then the Euclidean distance between $\overline{\overline{sp}}_1$ and $\overline{\overline{sp}}_2$ is defined as follows:

$$\overline{\overline{D}}(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2)=\sqrt{\frac{1}{2}\left(\left(\overline{\overline{po}}_1-\overline{\overline{po}}_2\right)^2+\left(\overline{\overline{ne}}_1-\overline{\overline{ne}}_2\right)^2+\left(\overline{\overline{fl}}_1-\overline{\overline{fl}}_2\right)^2\right)} \tag{8}$$

Power average operator

PA operator was initially developed by Yager (2001) for crisp number. The main advantage of the PA operator is its capacity to diminish the inadequate effect of unreasonably high and low arguments on the final results.

Definition 8 (Yager 2001) Let $\overline{\overline{a}}_l(1,\ldots,q)$ be a group of crisp numbers, then the PA operator is represented as follows:

$$PA\left(\overline{\overline{a}}_1,\overline{\overline{a}}_2,\ldots,\overline{\overline{a}}_q\right)=\sum_{l=1}^{q}\left(\frac{\left(1+T\left(\overline{\overline{a}}_l\right)\right)\overline{\overline{a}}_l}{\sum_{m=1}^{q}\left(1+T\left(\overline{\overline{a}}_m\right)\right)}\right), \tag{9}$$

where $T\left(\overline{\overline{a}}_m\right)=\sum_{\substack{n=1,\\ m\neq n}}^{q} Sup\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)$ and $Sup\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)$ is the support degree for $\overline{\overline{a}}_m$ and $\overline{\overline{a}}_n$. The support must satisfy the following axioms:

1. $Sup\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)\in[0,1]$;
2. $Sup\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)=Sup\left(\overline{\overline{a}}_n,\overline{\overline{a}}_m\right)$;
3. If $\overline{\overline{D}}\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)<\overline{\overline{D}}\left(\overline{\overline{a}}_u,\overline{\overline{a}}_v\right)$ then $Sup\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)>Sup\left(\overline{\overline{a}}_u,\overline{\overline{a}}_v\right)$, where $\overline{\overline{D}}\left(\overline{\overline{a}}_m,\overline{\overline{a}}_n\right)$ is the distance measure among $\overline{\overline{a}}_m$ and $\overline{\overline{a}}_n$.

Dombi t-norm and Dombi t-conorm

Definition 9 Dombi 1982) Assume that $(\overline{\overline{t}},\overline{\overline{s}})\in(0,1)\times(0,1)$ are any real numbers with $\gamma\geq 1$. Then, Dombi t-norm and Dombi t-conorm are described as follows:

$$\overline{\overline{T}}\left(\overline{\overline{t}},\overline{\overline{s}}\right)=\frac{1}{1+\left\{\left(\frac{1-\overline{\overline{t}}}{\overline{\overline{t}}}\right)^{\gamma}+\left(\frac{1-\overline{\overline{s}}}{\overline{\overline{s}}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}; \tag{10}$$

$$\overline{\overline{S}}\left(\overline{\overline{t}},\overline{\overline{s}}\right)=\frac{1}{1+\left\{\left(\frac{\overline{\overline{t}}}{1-\overline{\overline{t}}}\right)^{\gamma}+\left(\frac{\overline{\overline{s}}}{1-\overline{\overline{s}}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}. \tag{11}$$

Improved Dombi operational laws for SPFNs

In this section, we aim to develop the improved Dombi aggregation operators for SPFBNs. We give examples to support our proposed work.

Existing Dombi operational laws for SPFNs

In this subsection, we discuss limitations in the existing Dombi operational laws proposed by Ashraf et al. (2019b).

Definition 10 (Ashraf et al. 2019b) Let $\overline{\overline{sp}}=\left\langle\overline{\overline{po}},\overline{\overline{ne}},\overline{\overline{fl}}\right\rangle$, $\overline{\overline{sp}}_1=\left\langle\overline{\overline{po}}_1,\overline{\overline{ne}}_1,\overline{\overline{fl}}_1\right\rangle$ and $\overline{\overline{sp}}_2=\left\langle\overline{\overline{po}}_2,\overline{\overline{ne}}_2,\overline{\overline{fl}}_2\right\rangle$ be any SPFNs, then Ashraf et al. (2019b) defined the Dombi operational laws for SPFNs as follows:

(1)
$$\overline{\overline{sp}}_1\oplus\overline{\overline{sp}}_2=\left\langle\sqrt{1-\frac{1}{1+\left\{\left(\frac{1}{\overline{\overline{po}}_1^2}-1\right)^{2\gamma}+\left(\frac{1}{\overline{\overline{po}}_2^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}},\ \frac{1}{\sqrt{1+\left\{\left(\frac{1}{\overline{\overline{ne}}_1^2}-1\right)^{2\gamma}+\left(\frac{1}{\overline{\overline{ne}}_2^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}},\ \frac{1}{\sqrt{1+\left\{\left(\frac{1}{\overline{\overline{fl}}_1^2}-1\right)^{2\gamma}+\left(\frac{1}{\overline{\overline{fl}}_2^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle; \tag{12}$$

(2)
$$\overline{\overline{sp}}_1\otimes\overline{\overline{sp}}_2=\left\langle\frac{1}{\sqrt{1+\left\{\left(\frac{1}{\overline{\overline{po}}_1^2}-1\right)^{2\gamma}+\left(\frac{1}{\overline{\overline{po}}_2^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}},\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{1}{\overline{\overline{ne}}_1^2}-1\right)^{2\gamma}+\left(\frac{1}{\overline{\overline{ne}}_2^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}},\ \sqrt{1-\frac{1}{1+\left\{\left(\frac{1}{\overline{\overline{fl}}_1^2}-1\right)^{2\gamma}+\left(\frac{1}{\overline{\overline{fl}}_2^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle; \tag{13}$$

(3)
$$\beta\overline{\overline{sp}}=\left\langle\sqrt{1-\frac{1}{1+\left\{\beta\left(\frac{1}{\overline{\overline{pe}}^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}},\ \frac{1}{\sqrt{1+\left\{\beta\left(\frac{1}{\overline{\overline{ne}}^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}},\ \frac{1}{\sqrt{1+\left\{\beta\left(\frac{1}{\overline{\overline{fl}}^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;\beta>0. \tag{14}$$

(4) $$\overline{\overline{sp}}^{\beta} = \left\langle \frac{1}{\sqrt{1+\left\{\beta\left(\frac{1}{\overline{\overline{po}}^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\beta\left(\frac{1}{\overline{\overline{ne}}^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\beta\left(\frac{1}{\overline{\overline{fl}}^2}-1\right)^{2\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \beta > 0. \tag{15}$$

In the above Dombi operational laws, there exist some limitations in the sum and multiplication operations which are discussed below:

Example 1 Let $\overline{\overline{sp}}_1 = \langle 0.4, 0.5, 0.5\rangle$ and $\overline{\overline{sp}}_2 = \langle 0.2, 0.6, 0.3\rangle$ be two SPFNs, then by utilizing (12), we have $\overline{\overline{sp}}_1 \oplus \overline{\overline{sp}}_2 = \langle 0.9991, 0.3080, 0.0982\rangle\ (\gamma = 2)$.

When we add the squares of this three positive-membership, neutral-membership, and negative-membership degrees, it becomes 1.1028, which is not an SPFN. Similarly, let $\overline{\overline{sp}}_1 = \langle 0.4, 0.3, 0.3\rangle$ and $\overline{\overline{sp}}_2 = \langle 0.6, 0.2, 0.2\rangle$ be two SPFNs, then by utilizing Eq. (13), we have

$\overline{\overline{sp}}_1 \otimes \overline{\overline{sp}}_2 = \langle 0.1865, 0.9991, 0.9991\rangle (\gamma = 2)$.

When we add the squares of these three positive-membership, neutral-membership, and negative-membership degrees, it becomes 2.0314, which is not an SPFN.

Hence, there is a need to improve the above Dombi operational laws for SPFNs. So, in the next subsection, we develop improved Dombi operational laws for SPFNs.

Improved Dombi operational laws for SPFNs

In this subsection, we propose some new improved operational laws for SPFNs.

Definition 11 Let $\overline{\overline{sp}} = \left\langle \overline{\overline{po}}, \overline{\overline{ne}}, \overline{\overline{fl}} \right\rangle$, $\overline{\overline{sp}}_1 = \left\langle \overline{\overline{po}}_1, \overline{\overline{ne}}_1, \overline{\overline{fl}}_1 \right\rangle$ and $\overline{\overline{sp}}_2 = \left\langle \overline{\overline{po}}_2, \overline{\overline{ne}}_2, \overline{\overline{fl}}_2 \right\rangle$ be any SPFNs, then the improved operational laws for SPFNs based on Dombi t-norm and Dombi t-conorm are defined as follows:

(1) $$\overline{\overline{sp}}_1 \oplus \overline{\overline{sp}}_2 = \left\langle \sqrt{1-\frac{1}{1+\left\{\left(\frac{\overline{\overline{po}}_1^2}{1-\overline{\overline{po}}_1^2}\right)^{\gamma}+\left(\frac{\overline{\overline{po}}_2^2}{1-\overline{\overline{po}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{\left(\frac{1-\overline{\overline{ne}}_1^2}{\overline{\overline{ne}}_1^2}\right)^{\gamma}+\left(\frac{1-\overline{\overline{ne}}_2^2}{\overline{\overline{ne}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{\left(\frac{1-\overline{\overline{fl}}_1^2}{\overline{\overline{fl}}_1^2}\right)^{\gamma}+\left(\frac{1-\overline{\overline{fl}}_2^2}{\overline{\overline{fl}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle; \tag{16}$$

(2) $$\overline{\overline{sp}}_1 \otimes \overline{\overline{sp}}_2 = \left\langle \sqrt{\frac{1}{1+\left\{\left(\frac{1-\overline{\overline{po}}_1^2}{\overline{\overline{po}}_1^2}\right)^{\gamma}+\left(\frac{1-\overline{\overline{po}}_2^2}{\overline{\overline{po}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\left(\frac{\overline{\overline{ne}}_1^2}{1-\overline{\overline{ne}}_1^2}\right)^{\gamma}+\left(\frac{\overline{\overline{ne}}_2^2}{1-\overline{\overline{ne}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\left(\frac{\overline{\overline{fl}}_1^2}{1-\overline{\overline{fl}}_1^2}\right)^{\gamma}+\left(\frac{\overline{\overline{fl}}_2^2}{1-\overline{\overline{fl}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle; \tag{17}$$

(3) $$\zeta\overline{\overline{sp}} = \left\langle \sqrt{1-\frac{1}{1+\left\{\zeta\left(\frac{\overline{\overline{po}}^2}{1-\overline{\overline{po}}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{\zeta\left(\frac{1-\overline{\overline{ne}}^2}{\overline{\overline{ne}}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1+\left\{\zeta\left(\frac{1-\overline{\overline{fl}}^2}{\overline{\overline{fl}}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \zeta > 0; \tag{18}$$

(4) $$\overline{\overline{sp}}^{\zeta} = \left\langle \sqrt{\frac{1}{1+\left\{\zeta\left(\frac{1-\overline{\overline{po}}^2}{\overline{\overline{po}}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\zeta\left(\frac{\overline{\overline{ne}}^2}{1-\overline{\overline{ne}}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{1-\frac{1}{1+\left\{\zeta\left(\frac{\overline{\overline{fl}}^2}{1-\overline{\overline{fl}}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle, \zeta > 0. \tag{19}$$

Now, reconsider Example 1, and by utilizing Eq. (16), we have

$\overline{\overline{sp}}_1 \oplus \overline{\overline{sp}}_2 = \langle 0.4039, 0.4721, 0.2943\rangle\ (\gamma = 2)$..

When we add the squares of these three positive-membership, neutral-membership, and negative-membership degrees, it becomes 0.6089, which is an SPFN. Similarly, let $\overline{\overline{sp}}_1 = \langle 0.4, 0.3, 0.3\rangle$ and $\overline{\overline{sp}}_2 = \langle 0.6, 0.2, 0.2\rangle$ be two SPFNs, then by utilizing Eq. (17), we have

$\overline{\overline{sp}}_1 \otimes \overline{\overline{sp}}_2 = \langle 0.3909, 0.3113, 0.3113\rangle (\gamma = 2)$.

When we add the squares of these three positive-membership, neutral-membership, and negative-membership degrees, it becomes 0.3467, which is an SPFN.

Dombi aggregation operators

In this section, firstly, we review the existing Dombi aggregation operators based on existing Dombi operations and discuss their limitations. Secondly, we develop improved Dombi aggregation operators based on improved Dombi operational laws.

Existing aggregation operators for SPFNs

In this subsection, we give some existing Dombi aggregation operators developed by Xu (2011) and discussed the limitations of these aggregation operators.

Definition 12 (Ashraf et al. 2019b) Let $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ be a group of SPFNs, then the spherical fuzzy Dombi weighted averaging (SPFDWA) operator is described as follows:

$$SPFDWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \sum_{h=1}^{g} w_h \overline{\overline{sp}}_h, \tag{20}$$

where $w = \left(w_1, w_2, \ldots, w_g\right)^T$ is weight vector with $w_h \in [0, 1]$ and $\sum_{h=1}^{g} w_h = 1$.

Theorem 1 *(Ashraf et al. 2019b) Let $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ be a group of SPFNs, then the structure of SPFWA operator is described utilizing Dombi operations with $\gamma > 0$;*

$$SPFDWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1}{\left(\overline{\overline{po}}_h\right)^2} - 1 \right)^{2\gamma} \right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1}{\left(\overline{\overline{ne}}_h\right)^2} - 1 \right)^{2\gamma} \right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1}{\left(\overline{\overline{fl}}_h\right)^2} - 1 \right)^{2\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle; \tag{21}$$

In some cases, when we aggregate SPFNs by (21), the obtained aggregated value is not an SPFN.

Example 2 Let $\overline{\overline{sp}}_1 = \langle 0.1, 0.6, 0.4 \rangle, \overline{\overline{sp}}_2 = \langle 0.2, 0.5, 0.4 \rangle, \overline{\overline{sp}}_3 = \langle 0.4, 0.5, 0.5 \rangle$ and $\overline{\overline{sp}}_4 = \langle 0.3, 0.5, 0.6 \rangle$ be any four SPFNs and $w = (0.25, 0.4, 0.2, 0.15)^T$ be the weight vector of these four SPFNs. Then, by utilizing Eq. (21), we have ($\gamma = 2$)

$SPFDWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \overline{\overline{sp}}_3, \overline{\overline{sp}}_4\right) = \langle 0.9999, 0.3342, 0.2058 \rangle.$

Now, if the square of positive-membership, neutral-membership, and negative-membership degrees of the aggregated value is added, then its value is 1.1538. So it violates the condition that the sum of the square of positive-membership, neutral-membership, and negative-membership degrees is less or equal to 1. Therefore, the aggregated value is not an SPFN. Similar limitations exist in other structures such as spherical fuzzy Dombi ordered weighted averaging (SPFDOWA) operator and spherical fuzzy Dombi hybrid weighted averaging (SPFDHWA) operator.

Definition 13 (Ashraf et al. 2019b) Let $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ be a group of SPFNs, then the spherical fuzzy Dombi weighted geometric (SPFDWG) operator is described as follows:

$$SPFDWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \prod_{h=1}^{g} \left(\overline{\overline{sp}}_h\right)^{w_h}, \tag{22}$$

where $w = \left(w_1, w_2, \ldots, w_g\right)^T$ is weight vector with $w_h \in [0, 1]$ and $\sum_{h=1}^{g} w_h = 1$.

Theorem 2 *(Ashraf et al. 2019b) Let $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ be a group of SPFNs, then the structure of SPFWG operator is described by utilizing Dombi operations with $\gamma > 0$;*

$$SPFDWG\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \frac{1}{\sqrt{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1}{\left(\overline{\overline{po}}_h\right)^2} - 1 \right)^{2\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1}{\left(\overline{\overline{ne}}_h\right)^2} - 1 \right)^{2\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1}{\left(\overline{\overline{fl}}_h\right)^2} - 1 \right)^{2\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle; \tag{23}$$

In some cases, when we aggregate SPFNs utilizing Eq. (23), the obtained aggregated value is not an SPFN.

Example 3 Let $\overline{\overline{sp}}_1 = \langle 0.8, 0.1, 0.1 \rangle, \overline{\overline{sp}}_2 = \langle 0.7, 0.2, 0.3 \rangle, \overline{\overline{sy}}_3 = \langle 0.9, 0.2, 0.2 \rangle$ and $\overline{\overline{sp}}_4 = \langle 0.5, 0.3, 0.4 \rangle$ be any four SPFNs and $w = (0.25, 0.4, 0.2, 0.15)^T$ be the weight vector of these four SPFNs. Then, by utilizing Eq. (23), we have ($\gamma = 2$)

$SPFDWG\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \overline{\overline{sp}}_3, \overline{\overline{sp}}_4\right) = \langle 0.4685, 0.9999, 0.9999 \rangle.$

Now, if the square of positive-membership, neutral-membership, and negative-membership degrees of the aggregated value is added, then its value is 2.2191. So it violates the condition that the sum of squares of membership and nonmembership is less or equal to 1. Therefore, the aggregated value is not an SPFN. Similar limitations exist in other structures such as spherical fuzzy Dombi ordered weighted geometric (SPFDOWG) operator and spherical fuzzy Dombi hybrid weighted geometric (SPFDHWG) operator.

Improved Dombi aggregation operators for SPFNs

The definitions of the spherical Dombi weighted aggregation operators remain the same as those defined by Ashraf et al. (2019b). We just improved some theorems.

Theorem 3 *Let* $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *be a group of SPFNs, then the structure of improved SPFWA (ISPFWA) operator is described as follows by utilizing improved Dombi operations with* $\gamma > 0$*;*

$$ISPFDWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{\overline{\overline{po}}_h^2}{1 - \overline{\overline{po}}_h^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle. \tag{24}$$

Now considering Example 2 and aggregating those values by utilizing Eq. (24), we have ($\gamma = 2$).

$$ISPFDWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \overline{\overline{sp}}_3, \overline{\overline{sp}}_4\right) = \langle 0.2976, 0.5167, 0.4267 \rangle$$

;

Now, if the square of positive-membership, neutral-membership, and negative-membership degrees of the aggregated value is added, then its value is 0.5376. Hence, the obtained aggregated value is an SPFN.

Similarly, the improved structure of other Dombi AOs such as improved SPFDWOA (ISPFDWOA) operator and improved SPFDHWA (ISPFDHWA) operator is described as follows.

$$ISPFDOWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{\overline{\overline{po}}_{\sigma(h)}^2}{1 - \overline{\overline{po}}_{\sigma(h)}^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \overline{\overline{ne}}_{\sigma(h)}^2}{\overline{\overline{ne}}_{2\,\sigma(h)}} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \overline{\overline{fl}}_{\sigma(h)}^2}{\overline{\overline{fl}}_{2\,\sigma(h)}} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle. \tag{25}$$

$$ISPFDHWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{\left(\overline{\overline{po}}^{*}_{\sigma(h)}\right)^2}{1 - \left(\overline{\overline{po}}^{*}_{\sigma(h)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \left(\overline{\overline{ne}}^{*}_{\sigma(h)}\right)^2}{\left(\overline{\overline{ne}}^{*}_{\sigma(h)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \left(\overline{\overline{fl}}^{*}_{\sigma(h)}\right)^2}{\left(\overline{\overline{fl}}^{*}_{\sigma(h)}\right)^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle. \tag{26}$$

Theorem 4 *Let* $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *be a group of SPFNs, then the structure of improved SPFWG (ISPFWG) operator is described by utilizing improved Dombi operations with* $\gamma > 0$ *as*

$$ISPFDWG\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{1 - \overline{\overline{pe}}_h}{\overline{\overline{pe}}_h} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}, \sqrt{1 - \frac{1}{1 + \left\{ \sum_{h=1}^{g} w_h \left(\frac{\overline{\overline{ne}}_h^2}{1 - \overline{\overline{ne}}_h^2} \right)^{\gamma} \right\}^{\frac{1}{\gamma}}}} \right\rangle. \tag{27}$$

Now considering Example 3 and aggregating those values by utilizing Eq. (27), we have ($\gamma = 2$)

$$ISPFDWG\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \overline{\overline{sp}}_3, \overline{\overline{sp}}_4\right) = \langle 0.6497, 0.2189, 0.2996 \rangle.$$

Now, if the square of positive-membership, neutral-membership, and negative-membership degrees of the aggregated value is added, then its value is 0.5599. Hence, the obtained aggregated value is an SPFN.

Similarly, the improved structure of other Dombi aggregation operators such as the improved SPFDWOG (ISPFDWOG) operator and the improved SPFDHWG (ISPFDHWG) operator is described as follows.

$$ISPFDOWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-\overline{\overline{po}}_{\sigma(h)}^{2}}{\overline{\overline{po}}_{\sigma(h)}^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\overline{\overline{ne}}_{\sigma(h)}^{2}}{1-\overline{\overline{ne}}_{\sigma(h)}^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\overline{\overline{fl}}_{\sigma(h)}^{2}}{1-\overline{\overline{fl}}_{\sigma(h)}^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{28}$$

$$ISPFDHWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{1-\overline{\overline{po}}_{\sigma(h)}^{*2}}{\overline{\overline{po}}_{\sigma(h)}^{*2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\left(\overline{\overline{ne}}_{\sigma(h)}^{*}\right)^{2}}{1-\left(\overline{\overline{ne}}_{\sigma(h)}^{*}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}w_h\left(\frac{\left(\overline{\overline{fl}}_{\sigma(h)}^{*}\right)^{2}}{1-\left(\overline{\overline{fl}}_{\sigma(h)}^{*}\right)^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{29}$$

Spherical fuzzy Dombi power AOs

In this section, we develop two new Dombi AOs to deal with spherical fuzzy information. We divide this section into two subsections where the proposed arithmetic and geometric aggregation operators are briefly discussed.

Spherical fuzzy Dombi power arithmetic AOs

This section is about power arithmetic aggregation operators and their properties.

Definition 14 Let $\overline{\overline{sp}}_h\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ be a group of SPFNs, then we describe the SPFDPA operator as follows:

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\frac{\bigoplus_{h=1}^{g}\left(\left(1+T\left(\overline{\overline{sp}}_h\right)\right)\overline{\overline{sp}}_h\right)}{\sum_{h=1}^{g}\left(1+T\left(\overline{\overline{sp}}_h\right)\right)} \tag{30}$$

where

$$T\left(\overline{\overline{sp}}_h\right)=\sum_{\substack{k=1\\k\neq h}}^{g}Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right) \tag{31}$$

and $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)$ is the support for $\overline{\overline{sp}}_h$ from $\overline{\overline{sp}}_k$, which must satisfy the following conditions:

1. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\in[0,1]$;
2. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)=Sup\left(\overline{\overline{sp}}_k,\overline{\overline{sp}}_h\right)$;
3. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\geq Sup\left(\overline{\overline{sp}}_m,\overline{\overline{sp}}_n\right)$, if $\overline{\overline{D}}\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)<\overline{\overline{D}}\left(\overline{\overline{sp}}_m,\overline{\overline{sp}}_n\right)$, where $\overline{\overline{D}}$ is the distance measure given in Definition (7).

In order to write Eq. (30) in a simple form, we assume that

$$\Xi_h=\frac{\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}{\sum_{h=1}^{g}\left(1+T\left(\overline{\overline{sp}}_h\right)\right)} \tag{32}$$

Hence, Eq. (30) becomes

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\bigoplus_{h=1}^{g}\Xi_h\overline{\overline{sp}}_h \tag{33}$$

Based on Definition 14, we have the following result.

Theorem 5 *Let $\overline{\overline{sp}}_h\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ be a group of SPFNs, then the result aggregated by utilizing Eq. (30) is still SPFN, and*

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\left\langle\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}\Xi_h\left(\frac{\overline{\overline{pe}}_h^{2}}{1-\overline{\overline{pe}}_h^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^{2}}{\overline{\overline{ne}}_h^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^{2}}{\overline{\overline{fl}}_h^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{34}$$

Proof Based on Dombi operational laws, we have.

$$\Xi_h\overline{\overline{sp}}_h=\left\langle\sqrt{1-\frac{1}{1+\left\{\Xi_h\left(\frac{\overline{\overline{po}}_h^{2}}{1-\overline{\overline{po}}_h^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^{2}}{\overline{\overline{ne}}_h^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^{2}}{\overline{\overline{fl}}_h^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{35}$$

We prove Eq. (34) by utilizing mathematical induction. If $g=2$, then from Eq. (35), we have

$$\Xi_1\overline{\overline{sp}}_1=\left\langle\sqrt{1-\frac{1}{1+\left\{\Xi_1\left(\frac{\overline{\overline{po}}_1^{2}}{1-\overline{\overline{po}}_1^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_1\left(\frac{1-\overline{\overline{ne}}_1^{2}}{\overline{\overline{ne}}_1^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_1\left(\frac{1-\overline{\overline{fl}}_1^{2}}{\overline{\overline{fl}}_1^{2}}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,$$

and

$$\Xi_2\overline{\overline{sp}}_2=\left\langle\sqrt{1-\frac{1}{1+\left\{\Xi_2\left(\frac{\overline{\overline{po}}_2^2}{1-\overline{\overline{po}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_2\left(\frac{1-\overline{\overline{ne}}_2^2}{\overline{\overline{ne}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_2\left(\frac{1-\overline{\overline{fl}}_2^2}{\overline{\overline{fl}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,$$

So,

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2\right)=\Xi_1\overline{\overline{sp}}_1\oplus\Xi_2\overline{\overline{sp}}_2;$$

$$=\left\langle\sqrt{1-\frac{1}{1+\left\{\Xi_1\left(\frac{\overline{\overline{po}}_1^2}{1-\overline{\overline{po}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_1\left(\frac{1-\overline{\overline{ne}}_1^2}{\overline{\overline{ne}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_1\left(\frac{1-\overline{\overline{fl}}_1^2}{\overline{\overline{fl}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle\oplus\left\langle\sqrt{1-\frac{1}{1+\left\{\Xi_2\left(\frac{\overline{\overline{po}}_2^2}{1-\overline{\overline{po}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_2\left(\frac{1-\overline{\overline{ne}}_2^2}{\overline{\overline{ne}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_2\left(\frac{1-\overline{\overline{fl}}_2^2}{\overline{\overline{fl}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;$$

$$=\left\langle\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{\overline{\overline{po}}_h^2}{1-\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;$$

Therefore,

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2\right)=\left\langle\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{\overline{\overline{po}}_h^2}{1-\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{36}$$

Hence, Eq. (34) is true for $g=2$.

If $g=q$, by Eq. (34), we get the following equation:

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_q\right)=\left\langle\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{\overline{\overline{po}}_h^2}{1-\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{37}$$

If $g=q+1$, then based on Eqs. (36) and (37), we have the following formula:

$$SPFDPA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_q,\overline{\overline{sp}}_{q+1}\right)=\bigoplus_{h=1}^{q}\Xi_h\overline{\overline{sp}}_h\oplus\Xi_{q+1}\overline{\overline{sp}}_{q+1};$$

$$=\left\langle\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{\overline{\overline{po}}_h^2}{1-\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle$$

$$\oplus\left\langle\sqrt{1-\frac{1}{1+\left\{\Xi_{q+1}\left(\frac{\overline{\overline{po}}_{q+1}^2}{1-\overline{\overline{po}}_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_{q+1}\left(\frac{1-\overline{\overline{ne}}_{q+1}^2}{\overline{\overline{ne}}_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\Xi_{q+1}\left(\frac{1-\overline{\overline{fl}}_{q+1}^2}{\overline{\overline{fl}}_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;=\left\langle\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q+1}\Xi_h\left(\frac{\overline{\overline{po}}_h^2}{1-\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q+1}\Xi_h\left(\frac{1-\overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q+1}\Xi_h\left(\frac{1-\overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle.$$

Thus, Eq. (34) is true for g. Hence, Theorem 5 is true. The proof is completed. □

Definition 15 Let $\overline{\overline{sp}}_h\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ be a group of SPFNs, $\Theta=\left(\Theta_1,\Theta_2,\ldots,\Theta_g\right)^T$ be the weight vector of $\overline{\overline{sp}}_h(h=1,2,\ldots,g)$, such that $\Theta_h\in[0,1]$ and $\sum_{h=1}^{g}\Theta_h=1$. Then, the spherical fuzzy Dombi power weighted averaging (SPFDPWA) operator is described as follows:

$$SPFDPWA\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\frac{\bigoplus_{h=1}^{g}\left(\Theta_h\left(1+T\left(\overline{\overline{sp}}_h\right)\right)\overline{\overline{sp}}_h\right)}{\sum_{h=1}^{g}\Theta_h\left(1+T\left(\overline{\overline{sp}}_h\right)\right)} \tag{38}$$

where

$$T\left(\overline{\overline{sp}}_h\right)=\sum_{\substack{k=1\\k\neq h}}^{g}Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right) \tag{39}$$

and $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)$ is the support for $\overline{\overline{sp}}_h$ from $\overline{\overline{sp}}_k$, which must satisfy the following conditions:

1. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\in[0,1]$;
2. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)=Sup\left(\overline{\overline{sp}}_k,\overline{\overline{sp}}_h\right)$;
3. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\geq Sup\left(\overline{\overline{sp}}_m,\overline{\overline{sp}}_n\right)$, if $\overline{\overline{D}}\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)<\overline{\overline{D}}\left(\overline{\overline{sp}}_m,\overline{\overline{sp}}_n\right)$, where $\overline{\overline{D}}$ is the distance measure given in Definition (7).

In order to write Eq. (38) in simple form, we assume that

$$\Psi_h = \frac{\Theta_h\left(1 + T\left(\overline{\overline{sp}}_h\right)\right)}{\sum_{h=1}^{g} \Theta_h\left(1 + T\left(\overline{\overline{sp}}_h\right)\right)} \tag{40}$$

Hence, Eq. (36) becomes

$$SPFDPWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \bigoplus_{h=1}^{g} \Psi_h \overline{\overline{sp}}_h \tag{41}$$

Based on Definition 15, we have the following result.

Theorem 6 *Let* $\overline{\overline{sp}}_h\left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *be a group of SPFNs, then the result aggregated by utilizing Eq. (36) is still SPFN, and*

$$SPFDPWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \left\langle \sqrt{1 - \frac{1}{1 + \left\{\sum_{h=1}^{g} \Psi_h \left(\frac{\overline{\overline{po}}_h^2}{1-\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{\sum_{h=1}^{g} \Psi_h \left(\frac{1-\overline{\overline{ne}}_h^2}{\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt{\frac{1}{1 + \left\{\sum_{h=1}^{g} \Psi_h \left(\frac{1-\overline{\overline{fl}}_h^2}{\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}} \right\rangle. \tag{42}$$

Proof Proof of Theorem 6 is the same as Theorem 5, omitted here. □

It can be easily proved that the SPFDPWA operator has the following properties:

Property 1 (Idempotency) *If all* $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *are equal, that is* $\overline{\overline{sp}}_h = \overline{\overline{sp}}$ *for all h. Then,*

$$SPFDPWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \overline{\overline{sp}}. \tag{43}$$

Property 2 (Boundedness) *Let* $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *be a group of SPFNs, and let* $\overline{\overline{sp}}^- = \min_h \overline{\overline{sp}}_h$, $\overline{\overline{sp}}^+ = \max_h \overline{\overline{sp}}_h$. *Then*

$$\overline{\overline{sp}}^- \le SPFDPWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) \le \overline{\overline{sp}}^+. \tag{44}$$

Property 3 (Monotonicity) *Let* $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *and* $\overline{\overline{sp}}_h^{\sim} = \left\langle \overline{\overline{po}}_h^{\sim}, \overline{\overline{ne}}_h^{\sim}, \overline{\overline{fl}}_h^{\sim} \right\rangle (h = 1, \ldots, g)$ *be two groups of SPFNs, if* $\overline{\overline{sp}}_h \le \overline{\overline{sp}}_h^{\sim}$ *for all h. Then*

$$SPFDPWA\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) \le SPFDPWA\left(\overline{\overline{sp}}_1^{\sim}, \overline{\overline{sp}}_2^{\sim}, \ldots, \overline{\overline{sp}}_g^{\sim}\right). \tag{45}$$

Spherical fuzzy Dombi power geometric aggregation (SPFDPGA) operators

In this section, the conception of Dombi power geometric aggregation operators is investigated with the help of examples.

Definition 16 Let $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ be a group of SPFNs, then we describe the SPFDPG operator as follows:

$$SPFDPG\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \bigotimes_{h=1}^{g} \left(\overline{\overline{sp}}_h\right)^{\frac{\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}{\sum_{h=1}^{g}\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}} \tag{46}$$

where

$$T\left(\overline{\overline{sp}}_h\right) = \sum_{\substack{k=1 \\ k \ne h}}^{g} Sup\left(\overline{\overline{sp}}_h, \overline{\overline{sp}}_k\right) \tag{47}$$

and $Sup\left(\overline{\overline{sp}}_h, \overline{\overline{sp}}_k\right)$ is the support for $\overline{\overline{sp}}_h$ from $\overline{\overline{sp}}_k$, which must satisfy the following conditions:

1. $Sup\left(\overline{\overline{sp}}_h, \overline{\overline{sp}}_k\right) \in [0, 1]$;
2. $Sup\left(\overline{\overline{sp}}_h, \overline{\overline{sp}}_k\right) = Sup\left(\overline{\overline{sp}}_k, \overline{\overline{sp}}_h\right)$;
3. $Sup\left(\overline{\overline{sp}}_h, \overline{\overline{sp}}_k\right) \ge Sup\left(\overline{\overline{sp}}_m, \overline{\overline{sp}}_n\right)$, if $\overline{\overline{D}}\left(\overline{\overline{sp}}_h, \overline{\overline{sp}}_k\right) < \overline{\overline{D}}\left(\overline{\overline{sp}}_m, \overline{\overline{sp}}_n\right)$, where $\overline{\overline{D}}$ is the distance measure given in Definition (7).

In order to write Eq. (46) in a simple form, we assume that

$$\Xi_h = \frac{\left(1 + T\left(\overline{\overline{sp}}_h\right)\right)}{\sum_{h=1}^{g}\left(1 + T\left(\overline{\overline{sp}}_h\right)\right)} \tag{48}$$

Hence, Eq. (46) becomes

$$SPFDPG\left(\overline{\overline{sp}}_1, \overline{\overline{sp}}_2, \ldots, \overline{\overline{sp}}_g\right) = \bigotimes_{h=1}^{g} \left(\overline{\overline{sp}}_h\right)^{\Xi_h} \tag{49}$$

Based on Definition 16, we have the following result.

Theorem 7 *Let* $\overline{\overline{sp}}_h = \left\langle \overline{\overline{po}}_h, \overline{\overline{ne}}_h, \overline{\overline{fl}}_h \right\rangle (h = 1, \ldots, g)$ *be a group of SPFNs, then the result aggregated by utilizing Eq. (46) is still SPFN, and*

$$SPFDPG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}\Xi_h\left(\frac{1-\overline{\overline{pe}}_h^2}{\overline{\overline{pe}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{50}$$

Proof Based on Dombi operational laws, we have

$$\left(\overline{\overline{sp}}_h\right)^{\Xi_h}=\left\langle\sqrt{\frac{1}{1+\left\{\Xi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{51}$$

We prove Eq. (50) by utilizing mathematical induction. If $g=2$, then from Eq. (51), we have

$$\left(\overline{\overline{sp}}_1\right)^{\Xi_1}=\left\langle\sqrt{\frac{1}{1+\left\{\Xi_1\left(\frac{1-\overline{\overline{po}}_1^2}{\overline{\overline{po}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_1\left(\frac{\overline{\overline{ne}}_1^2}{1-\overline{\overline{ne}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_1\left(\frac{\overline{\overline{fl}}_1^2}{1-\overline{\overline{fl}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,$$

and

$$\left(\overline{\overline{sp}}_2\right)^{\Xi_2}=\left\langle\sqrt{\frac{1}{1+\left\{\Xi_2\left(\frac{1-\overline{\overline{po}}_2^2}{\overline{\overline{po}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_2\left(\frac{\overline{\overline{ne}}_2^2}{1-\overline{\overline{ne}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_2\left(\frac{\overline{\overline{fl}}_2^2}{1-\overline{\overline{fl}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle,$$

So,

$$SPFDPG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2\right)=\left(\overline{\overline{sp}}_1\right)^{\Xi_1}\otimes\left(\overline{\overline{sp}}_2\right)^{\Xi_2};$$

$$=\left\langle\sqrt{\frac{1}{1+\left\{\Xi_1\left(\frac{1-\overline{\overline{po}}_1^2}{\overline{\overline{po}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_1\left(\frac{\overline{\overline{ne}}_1^2}{1-\overline{\overline{ne}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_1\left(\frac{\overline{\overline{fl}}_1^2}{1-\overline{\overline{fl}}_1^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle$$

$$\otimes\left\langle\sqrt{\frac{1}{1+\left\{\Xi_2\left(\frac{1-\overline{\overline{po}}_2^2}{\overline{\overline{po}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_2\left(\frac{\overline{\overline{ne}}_2^2}{1-\overline{\overline{ne}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_2\left(\frac{\overline{\overline{fl}}_2^2}{1-\overline{\overline{fl}}_2^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;$$

$$=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;$$

Therefore,

$$SPFDPG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2\right)=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{2}\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{52}$$

Hence, Eq. (50) is true for $g=2$.

If $g=q$, by Eq. (50), we get the following equation:

$$SPFDPG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_q\right)=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle. \tag{53}$$

If $g=q+1$, then based on Eqs. (52) and (53), we have the following formula:

$$SPFDPG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_q,\overline{\overline{sp}}_{q+1}\right)=\underset{h=1}{\overset{q}{\otimes}}\left(\overline{\overline{sp}}_h\right)^{\Xi_h}\otimes\left(\overline{\overline{sp}}_{q+1}\right)^{\Xi_{q+1}};$$
$$=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q}\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle$$
$$\otimes\left\langle\sqrt{\frac{1}{1+\left\{\Xi_{q+1}\left(\frac{1-\overline{\overline{po}}_{q+1}^2}{\overline{\overline{po}}_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_{q+1}\left(\frac{\overline{\overline{ne}}_{q+1}^2}{1-\overline{\overline{ne}}_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\Xi_{q+1}\left(\frac{\overline{\overline{fl}}_{q+1}^2}{1-\overline{\overline{fl}}_{q+1}^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle;$$
$$=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{q+1}\Xi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q+1}\Xi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{q+1}\Xi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle$$

Thus, Eq. (50) is true for g. Hence, Theorem 7 is true. The proof is completed. □

Definition 17 Let $\overline{\overline{sp}}_h=\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ be a group of SPFNs, $\Theta=\left(\Theta_1,\Theta_2,\ldots,\Theta_g\right)^T$ be the weight vector of $\overline{\overline{sp}}_h(h=1,2,\ldots,g)$, such that $\Theta_h\in[0,1]$ and $\sum_{h=1}^{g}\Theta_h=1$. Then, the spherical fuzzy Dombi power weighted geometric (SPFDPWG) operator is described as follows:

$$SPFDPWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\underset{h=1}{\overset{g}{\otimes}}\left(\overline{\overline{sp}}_h\right)^{\frac{\Theta_h\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}{\sum_{h=1}^{g}\Theta_h\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}}\tag{54}$$

where

$$T\left(\overline{\overline{sp}}_h\right)=\sum_{\substack{k=1\\k\neq h}}^{g}Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\tag{55}$$

and $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)$ is the support for $\overline{\overline{sp}}_h$ from $\overline{\overline{sp}}_k$, which must satisfy the following conditions:

1. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\in[0,1]$;
2. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)=Sup\left(\overline{\overline{sp}}_k,\overline{\overline{sp}}_h\right)$;
3. $Sup\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)\geq Sup\left(\overline{\overline{sp}}_m,\overline{\overline{sp}}_n\right)$, if $\overline{\overline{D}}\left(\overline{\overline{sp}}_h,\overline{\overline{sp}}_k\right)<\overline{\overline{D}}\left(\overline{\overline{sp}}_m,\overline{\overline{sp}}_n\right)$, where $\overline{\overline{D}}$ is the distance measure given in Definition (7).

In order to write Eq. (54) in a simple form, we assume that

$$\Psi_h=\frac{\Theta_h\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}{\sum_{h=1}^{g}\Theta_h\left(1+T\left(\overline{\overline{sp}}_h\right)\right)}\tag{56}$$

Hence, Eq. (54) becomes

$$SPFDPWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\underset{h=1}{\overset{g}{\otimes}}\left(\overline{\overline{sp}}_h\right)^{\Psi_h}\tag{57}$$

Based on Definition (17), we have the following result.

Theorem 8 *Let* $\overline{\overline{sp}}_h=\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ *be a group of SPFNs, then the result aggregated by utilizing Eq. (52) is still SPFN, and*

$$SPFDPWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{h=1}^{g}\Psi_h\left(\frac{1-\overline{\overline{po}}_h^2}{\overline{\overline{po}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}\Psi_h\left(\frac{\overline{\overline{ne}}_h^2}{1-\overline{\overline{ne}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{h=1}^{g}\Psi_h\left(\frac{\overline{\overline{fl}}_h^2}{1-\overline{\overline{fl}}_h^2}\right)^{\gamma}\right\}^{\frac{1}{\gamma}}}}\right\rangle.\tag{58}$$

Proof Proof of Theorem 8 is the same as Theorem 7, omitted here. □

It can be easily proved that the SPFDPWA operator has the following properties.

Property 1 (Idempotency) *If all* $\overline{\overline{sp}}_h=\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ *are equal, that is* $\overline{\overline{sp}}_h=\overline{\overline{sp}}$ *for all h, then*

$$SPFDPWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)=\overline{\overline{sp}}.\tag{59}$$

Property 2 (Boundedness) *Let* $\overline{\overline{sp}}_h=\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ *be a group of SPFNs, and let* $\overline{\overline{sp}}^-=\min_h\overline{\overline{sp}}_h$, $\overline{\overline{sp}}^+=\max_h\overline{\overline{sp}}_h$, *then*

$$\overline{\overline{sp}}^-\leq SPFDPWG\left(\overline{\overline{sy}}_1,\overline{\overline{sy}}_2,\ldots,\overline{\overline{sy}}_g\right)\leq\overline{\overline{sp}}^+.\tag{60}$$

Property 3 (Monotonicity) *Let* $\overline{\overline{sp}}_h=\left\langle\overline{\overline{po}}_h,\overline{\overline{ne}}_h,\overline{\overline{fl}}_h\right\rangle(h=1,\ldots,g)$ *and* $\overline{\overline{sp}}_h^{\sim}=\left\langle\overline{\overline{po}}_h^{\sim},\overline{\overline{ne}}_h^{\sim},\overline{\overline{fl}}_h^{\sim}\right\rangle(h=1,\ldots,g)$ *be two groups of SPFNs, if* $\overline{\overline{sp}}_h\leq\overline{\overline{sp}}_h^{\sim}$ *for all h, then*

$$SPFDPWG\left(\overline{\overline{sp}}_1,\overline{\overline{sp}}_2,\ldots,\overline{\overline{sp}}_g\right)\leq SPFDPWG\left(\overline{\overline{sp}}_1^{\sim},\overline{\overline{sp}}_2^{\sim},\ldots,\overline{\overline{sp}}_g^{\sim}\right).\tag{61}$$

Models for MAGDM with spherical fuzzy information

In this section, we shall employ the SPFDPWA operator and SPFDPWG operator to MAGDM with spherical fuzzy information. The following notions or statements are utilized to express the MAGDM problems for potential evaluation of emerging technology enterprises with spherical fuzzy information.

Let the set of discrete alternative be expressed by $\overline{\overline{AV}} = \left\{\overline{\overline{AV}}_1, \overline{\overline{AV}}_2, \ldots, \overline{\overline{AV}}_g\right\}$ and the set of attributes be represented by $\overline{\overline{AS}} = \left\{\overline{\overline{AS}}_1, \overline{\overline{AS}}_2, \ldots, \overline{\overline{AS}}_q\right\}$. Let the importance degree of the attributes be denoted by $\Theta = (\Theta_1, \Theta_2, \ldots, \Theta_q)^T$ such that $\Theta_p \in [0,1]$ and $\sum_{p=1}^{q} \Theta_p = 1$. There is a set of z experts expressed by $\overline{\overline{EX}} = \left\{\overline{\overline{EX}}_1, \overline{\overline{EX}}_2, \ldots, \overline{\overline{EX}}_z\right\}$, who are enquired to give the evaluation information, and the importance degree of the experts is expressed by $\Omega = (\Omega_1, \Omega_2, \ldots, \Omega_z)^T$, such that $\Omega_a \in [0,1], (a = 1,2,\ldots,z), \sum_{a=1}^{z} \Omega_a = 1$. The expert $\overline{\overline{EX}}_a$ assesses each attribute $\overline{\overline{AS}}_j$ of each alternative $\overline{\overline{AV}}_i$ by the form of SPFN $\overline{\overline{sp}}_{ij}^a = \left\langle \overline{\overline{po}}_{ij}^a, \overline{\overline{ne}}_{ij}^a, \overline{\overline{fl}}_{ij}^a \right\rangle (i = 1,2,\ldots,g;\ j = 1,2,\ldots,q)$, and then the decision matrices $\widetilde{DM}_a = \left(\overline{\overline{sp}}_{ij}^a\right)(a = 1,2,\ldots,z)$ is recognized. The subsequent purpose is to execute a ranking of all alternatives.

Then, in order to solve this problem, we will implement the following steps:

Step 1. Firstly, the given decision matrices $\widetilde{DM}_a = \left(\overline{\overline{sp}}_{ij}^a\right)_{m\times n}$ should be transformed into standardized decision matrices $\widetilde{SDM}_a = \left(\overline{\overline{sp}}_{ij}^a\right)_{m\times n}$. We change the cost-type attribute into a benefit-type attribute using the following formula.

$$\overline{\overline{sp}}_{ij}^a = \begin{cases} \overline{\overline{sp}}_{ij}^a = \left\langle \overline{\overline{po}}_{ij}^a, \overline{\overline{ne}}_{ij}^a, \overline{\overline{fl}}_{ij}^a \right\rangle & \text{for benefit} - \text{type attribute } \Xi_j \ _{i=1,2,\ldots,g, j=1,2,\ldots,q,} \\ \left(\overline{\overline{sp}}_{ij}^a\right)^c = \left\langle \overline{\overline{fl}}_{ij}^a, \overline{\overline{ne}}_{ij}^a, \overline{\overline{po}}_{ij}^a \right\rangle & \text{for benefit} - \text{type attribute } \Xi_j \end{cases} \tag{62}$$

Step 2. Determine the supports

$$Supp\left(\overline{\overline{sp}}_{ij}^c, \overline{\overline{sp}}_{ij}^d\right) = 1 - \overline{\overline{D}}\left(\overline{\overline{sp}}_{ij}^c, \overline{\overline{sp}}_{ij}^d\right), c, d = 1,2,\ldots,z \tag{63}$$

which fulfills the required axioms given in Definition 10, and $\overline{\overline{D}}\left(\overline{\overline{sp}}_{ij}^c, \overline{\overline{sp}}_{ij}^d\right)$ represents the distance measure given in Definition 7.

Step 3. Determine the supports $T\left(\overline{\overline{sp}}_{ij}^c\right)$ of the SPFN $\overline{\overline{sp}}_{ij}^c$ by other $\overline{\overline{sp}}_{ij}^d$ $(d = 1,2,\ldots,z \text{ and } c \neq d)$.

$$T\left(\overline{\overline{sp}}_{ij}^c\right) = \sum_{d=1; c\neq d}^{z} \Omega_d Sup\left(\overline{\overline{sp}}_{ij}^c, \overline{\overline{sp}}_{ij}^d\right);\ c, d = 1,2,\ldots z;\ i = 1,2,\ldots,g, j = 1,2,\ldots,q. \tag{64}$$

Then, use the importance degrees $\Omega_c (c = 1,2,\ldots,z)$ of the DMs $\Psi_a (a = 1,2,\ldots,z)$ to calculate the importance degrees

$$\varpi_{ij}^{(c)} = \frac{\Omega_c\left(1 + T\left(\overline{\overline{sp}}_{ij}^c\right)\right)}{\sum_{d}^{z} \Omega_d\left(1 + T\left(\overline{\overline{sp}}_{ij}^d\right)\right)};\ c = 1,2,\ldots,z;\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n. \tag{65}$$

where $\varpi_{ij} \geq 0$ and $\sum_{c=1}^{z} \varpi_{ij} = 1$.

Step 4. Utilize the SPDPWA operator or SPFDPWGA operator expressed in formula (42) and formula (58)

$$\overline{\overline{sp}}_{ij}^c = SPFDPWA\left(\overline{\overline{sp}}_{ij}^1, \overline{\overline{sp}}_{ij}^2, \ldots, \overline{\overline{sp}}_{ij}^z\right) \tag{66}$$

or

$$\overline{\overline{sp}}_{ij}^c = SPFDPWG\left(\overline{\overline{sp}}_{ij}^1, \overline{\overline{sp}}_{ij}^2, \ldots, \overline{\overline{sp}}_{ij}^z\right) \tag{67}$$

to aggregate all the decision matrices $\widetilde{DM}_c = \left(\overline{\overline{sp}}_{ij}^c\right)_{g\times q}$ $(c = 1,2,\ldots,z)$ given by the DMs into the comprehensive decision matrix $\widetilde{CDM} = \left(\overline{\overline{sp}}_{ij}\right)_{g\times q}$.

Step 5. Determine the supports:

$$Sup\left(\overline{\overline{sp}}_{ij}, \overline{\overline{sp}}_{il}\right) = 1 - \overline{\overline{D}}\left(\overline{\overline{sp}}_{ij}, \overline{\overline{sp}}_{il}\right),\ i = 1,2,\ldots,g;\ l = 1,2,\ldots,q. \tag{68}$$

Table 3 Spherical fuzzy decision matrix $\widetilde{DM}_1$ provided by expert $\overline{\overline{EX}}_1$

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.84, 0.34, 0.4\rangle$	$\langle 0.43, 0.39, 0.78\rangle$	$\langle 0.67, 0.45, 0.2\rangle$	$\langle 0.31, 0.21, 0.71\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.60, 0.32, 0.38\rangle$	$\langle 0.23, 0.35, 0.59\rangle$	$\langle 0.72, 0.31, 0.41\rangle$	$\langle 0.18, 0.20, 0.82\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.79, 0.19, 0.39\rangle$	$\langle 0.22, 0.25, 0.77\rangle$	$\langle 0.71, 0.41, 0.13\rangle$	$\langle 0.34, 0.23, 0.51\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.63, 0.51, 0.13\rangle$	$\langle 0.49, 0.33, 0.42\rangle$	$\langle 0.61, 0.43, 0.45\rangle$	$\langle 0.49, 0.37, 0.59\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.43, 0.41, 0.39\rangle$	$\langle 0.50, 0.45, 0.31\rangle$	$\langle 0.43, 0.32, 0.52\rangle$	$\langle 0.44, 0.44, 0.34\rangle$

Table 4 Spherical fuzzy decision matrix $\widetilde{DM}_2$ provided by expert $\overline{\overline{EX}}_2$

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.61, 0.15, 0.55\rangle$	$\langle 0.16, 0.35, 0.62\rangle$	$\langle 0.75, 0.25, 0.35\rangle$	$\langle 0.55, 0.17, 0.74\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.85, 0.22, 0.25\rangle$	$\langle 0.40, 0.20, 0.80\rangle$	$\langle 0.81, 0.15, 0.15\rangle$	$\langle 0.30, 0.20, 0.80\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.71, 0.19, 0.15\rangle$	$\langle 0.18, 0.15, 0.67\rangle$	$\langle 0.56, 0.17, 0.44\rangle$	$\langle 0.43, 0.13, 0.61\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.59, 0.32, 0.34\rangle$	$\langle 0.24, 0.48, 0.51\rangle$	$\langle 0.68, 0.53, 0.39\rangle$	$\langle 0.34, 0.21, 0.61\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.52, 0.23, 0.24\rangle$	$\langle 0.49, 0.44, 0.43\rangle$	$\langle 0.53, 0.44, 0.31\rangle$	$\langle 0.46, 0.49, 0.35\rangle$

Table 5 Spherical fuzzy decision matrix $\widetilde{DM}_3$ provided by expert $\overline{\overline{EX}}_3$

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.85, 0.25, 0.15\rangle$	$\langle 0.14, 0.23, 0.78\rangle$	$\langle 0.78, 0.38, 0.18\rangle$	$\langle 0.29, 0.39, 0.83\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.85, 0.1, 0.2\rangle$	$\langle 0.30, 0.19, 0.70\rangle$	$\langle 0.68, 0.12, 0.21\rangle$	$\langle 0.12, 0.23, 0.78\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.73, 0.13, 0.46\rangle$	$\langle 0.34, 0.39, 0.73\rangle$	$\langle 0.82, 0.35, 0.18\rangle$	$\langle 0.40, 0.16, 0.71\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.82, 0.12, 0.43\rangle$	$\langle 0.55, 0.21, 0.63\rangle$	$\langle 0.53, 0.33, 0.47\rangle$	$\langle 0.37, 0.32, 0.65\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.61, 0.24, 0.45\rangle$	$\langle 0.33, 0.41, 0.50\rangle$	$\langle 0.61, 0.44, 0.34\rangle$	$\langle 0.53, 0.45, 0.34\rangle$

Table 6 Normalized spherical fuzzy decision matrix $\widetilde{SDM}_1$

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.84, 0.34, 0.4\rangle$	$\langle 0.78, 0.39, 0.43\rangle$	$\langle 0.67, 0.45, 0.2\rangle$	$\langle 0.71, 0.21, 0.31\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.60, 0.32, 0.38\rangle$	$\langle 0.59, 0.35, 0.23\rangle$	$\langle 0.72, 0.31, 0.41\rangle$	$\langle 0.82, 0.2, 0.18\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.79, 0.19, 0.39\rangle$	$\langle 0.77, 0.25, 0.22\rangle$	$\langle 0.71, 0.41, 0.13\rangle$	$\langle 0.51, 0.23, 0.34\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.63, 0.51, 0.13\rangle$	$\langle 0.42, 0.33, 0.49\rangle$	$\langle 0.61, 0.43, 0.45\rangle$	$\langle 0.59, 0.37, 0.49\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.43, 0.41, 0.39\rangle$	$\langle 0.31, 0.45, 0.5\rangle$	$\langle 0.43, 0.32, 0.52\rangle$	$\langle 0.34, 0.44, 0.44\rangle$

which fulfills the required axioms given in Definition 10, and $\overline{\overline{D}}(\overline{\overline{sp}}_{ij}, \overline{\overline{sp}}_{il})$ represents the distance measure given in Definition 7.

Step 6. Determine the supports $T(\overline{\overline{sp}}_{ij})$ of the SPFN $\overline{\overline{sp}}_{ij}\,(i = 1, 2, \ldots, g;\ j = 1, 2, \ldots, q)$ by the importance degrees Υ_j of the attributes $\overline{\overline{AS}}_j$ and the importance degrees ϕ_{ij} that are associated with the SPFN $\overline{\overline{sp}}_{ij}$ by the importance degree Υ_j of the attributes $\overline{\overline{AS}}_j$.

Table 7 Normalized spherical fuzzy decision matrix $\widetilde{SDM}_2$

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.61, 0.15, 0.55\rangle$	$\langle 0.62, 0.35, 0.16\rangle$	$\langle 0.75, 0.25, 0.35\rangle$	$\langle 0.74, 0.17, 0.55\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.85, 0.22, 0.25\rangle$	$\langle 0.8, 0.2, 0.4\rangle$	$\langle 0.81, 0.15, 0.15\rangle$	$\langle 0.8, 0.2, 0.3\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.71, 0.19, 0.15\rangle$	$\langle 0.67, 0.15, 0.18\rangle$	$\langle 0.56, 0.17, 0.44\rangle$	$\langle 0.61, 0.13, 0.43\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.59, 0.32, 0.34\rangle$	$\langle 0.51, 0.48, 0.24\rangle$	$\langle 0.68, 0.53, 0.39\rangle$	$\langle 0.61, 0.21, 0.34\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.52, 0.23, 0.24\rangle$	$\langle 0.43, 0.44, 0.49\rangle$	$\langle 0.53, 0.44, 0.31\rangle$	$\langle 0.35, 0.49, 0.46\rangle$

Table 8 Normalized spherical fuzzy decision matrix $\widetilde{SDM}_3$

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.85, 0.25, 0.15\rangle$	$\langle 0.78, 0.23, 0.14\rangle$	$\langle 0.78, 0.38, 0.18\rangle$	$\langle 0.83, 0.39, 0.29\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.85, 0.1, 0.2\rangle$	$\langle 0.7, 0.19, 0.3\rangle$	$\langle 0.68, 0.12, 0.21\rangle$	$\langle 0.78, 0.23, 0.12\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.73, 0.13, 0.46\rangle$	$\langle 0.73, 0.39, 0.24\rangle$	$\langle 0.82, 0.35, 0.18\rangle$	$\langle 0.71, 0.16, 0.4\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.82, 0.12, 0.43\rangle$	$\langle 0.63, 0.21, 0.55\rangle$	$\langle 0.53, 0.33, 0.47\rangle$	$\langle 0.65, 0.32, 0.37\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.61, 0.24, 0.45\rangle$	$\langle 0.5, 0.41, 0.33\rangle$	$\langle 0.61, 0.44, 0.34\rangle$	$\langle 0.34, 0.45, 0.53\rangle$

$$T\left(\overline{\overline{sp}}_{ij}\right) = \sum_{l=1;l\neq j}^{q} \Upsilon_j Sup\left(\overline{\overline{sp}}_{ij}, \overline{\overline{sp}}_{il}\right),\ i = 1,2,\ldots,g,\ j,l = 1,2,\ldots,q. \tag{69}$$

$$\Theta_{ij} = \frac{\Upsilon_c\left(1 + T\left(\overline{\overline{sp}}_{ij}\right)\right)}{\sum_{j=1}^{n}\Upsilon_j\left(1 + T\left(\overline{\overline{sp}}_{ij}\right)\right)};\ i = 1,2,\ldots,g,\ j = 1,2,\ldots,q. \tag{70}$$

where $\Theta_{ij} \geq 0$ and $\sum_{c=1}^{z}\Theta_{ij} = 1$.

Step 7. Utilize the SPFDPWA operator or SPFDPWG operator given in formula (42) and formula (58):

$$\overline{\overline{sp}}_{ij} = SPFDPWA\left(\overline{\overline{sp}}_{i1}, \overline{\overline{sp}}_{i2}, \ldots, \overline{\overline{sp}}_{iq}\right) \tag{71}$$

Or

$$\overline{\overline{sp}}_{ij} = SPFDPWG\left(\overline{\overline{sp}}_{i1}, \overline{\overline{sp}}_{i2}, \ldots, \overline{\overline{sp}}_{iq}\right) \tag{72}$$

to get the comprehensive evaluation value.

Step 8. Determine the score and accuracy value of each SPFN $\overline{\overline{sp}}_i (i = 1,2,\ldots,n)$ using Definition 5.

Step 9. Rank all the alternatives and select the best one using Definition 6.

Numerical examples

In this section, we give numerical examples based on spherical fuzzy Dombi aggregation operators and discussed the results. Further, we also show the variation in variable parameter and its effectiveness on decision results.

Numerical example

In this section, we shall give a numerical example adapted from (Jin et al. 2019) to illustrate the effectiveness and practicality of the developed MAGDM model under spherical fuzzy information.

Suppose that there is a group with five accessible emerging technologies enterprises (alternatives) $\overline{\overline{AV}}_h (h = 1, \ldots, 5)$ to be selected. These five alternatives are as follows:

(1) Augmented reality $\overline{\overline{AV}}_1$; (2) personalized medicine $\overline{\overline{AV}}_2$; (3) artificial intelligence $\overline{\overline{AV}}_3$; (4) gene drive $\overline{\overline{AV}}_4$; (5) quantum computing $\overline{\overline{AV}}_5$. Three experts are invited with weight vector $\Psi = (0.314, 0.355, 0.331)^T$ to evaluate the five alternatives concerning the following four attributes (1) technical advancement $\overline{\overline{AS}}_1$, (2) potential market and market risk $\overline{\overline{AS}}_2$, (3) human resources, financial infrastructure, and industrialization infrastructure $\overline{\overline{AS}}_3$, and (4) progress of science and technology and employment creation $\overline{\overline{AS}}_4$ with vector $\Upsilon = (0.257, 0.247, 0.244, 0.252)^T$. The three experts provide their assessment values in the form of spherical fuzzy numbers given in Tables 3, 4 and 5.

To select the best emerging technology enterprise, the following steps are involved:

Step 1. Firstly, normalize the decision matrices. Since $\overline{\overline{AS}}_1$, $\overline{\overline{AS}}_2$ are of benefit-type criteria and $\overline{\overline{AS}}_3$, $\overline{\overline{AS}}_4$ are of cost-type criteria. So, we need to transform the cost-type

Table 9 Overall spherical fuzzy decision matrix utilizing SPDPWA

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.8181, 0.1853, 0.1949\rangle$	$\langle 0.7529, 0.2828, 0.0007\rangle$	$\langle 0.7467, 0.3022, 0.2086\rangle$	$\langle 0.7823, 0.2011, 0.3286\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.8258, 0.1302, 0.2383\rangle$	$\langle 0.7428, 0.2125, 0.0058\rangle$	$\langle 0.7610, 0.1447, 0.1838\rangle$	$\langle 0.8016, 0.2078, 0.1505\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.7491, 0.1570, 0.1893\rangle$	$\langle 0.7172, 0.1870, 0.0017\rangle$	$\langle 0.7972, 0.2111, 0.1642\rangle$	$\langle 0.6442, 0.1528, 0.3807\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.7518, 0.1551, 0.1703\rangle$	$\langle 0.5577, 0.2630, 0.0080\rangle$	$\langle 0.6283, 0.3907, 0.4269\rangle$	$\langle 0.6197, 0.2552, 0.3759\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.5462, 0.2543, 0.2953\rangle$	$\langle 0.4425, 0.4310, 0.0244\rangle$	$\langle 0.5472, 0.3750, 0.3497\rangle$	$\langle 0.3436, 0.4582, 0.4695\rangle$

criteria into benefit-type criteria utilizing formula (62). Hence, the normalized decision matrices are given in Tables 6, 7 and 8.

Step 2. Determine the supports $Sup\left(\overline{\overline{sp}}^{r}_{hp}, \overline{\overline{sp}}^{s}_{hl}\right)$ by utilizing formula (63). For computational simplicity, we shall denote the support by S^{fl}_{hp} instead of

$$Sup\left(\overline{\overline{sp}}^{f}_{hp}, \overline{\overline{sp}}^{l}_{hp}\right)(h = 1,\ldots,5, p = 1,\ldots 4, f, l = 1,\ldots,3).$$

$$S^{12}_{11} = S^{21}_{11} = 0.2361, S^{13}_{11} = S^{31}_{11} = 0.1880, S^{23}_{11} = S^{32}_{11} = 0.3373,$$
$$S^{12}_{12} = S^{21}_{12} = 0.2237, S^{13}_{12} = S^{31}_{12} = 0.2342, S^{23}_{12} = S^{23}_{12} = 0.1421,$$
$$S^{12}_{13} = S^{21}_{13} = 0.1856, S^{13}_{13} = S^{31}_{13} = 0.0933, S^{23}_{13} = S^{32}_{13} = 0.1528,$$
$$S^{12}_{14} = S^{21}_{14} = 0.1733, S^{13}_{14} = S^{31}_{14} = 0.1536, S^{23}_{14} = S^{23}_{14} = 0.2491,$$
$$S^{12}_{21} = S^{21}_{21} = 0.2114, S^{13}_{21} = S^{31}_{21} = 0.2677, S^{23}_{21} = S^{32}_{21} = 0.0919,$$
$$S^{12}_{22} = S^{21}_{22} = 0.2185, S^{13}_{22} = S^{31}_{22} = 0.1459, S^{23}_{22} = S^{23}_{22} = 0.1002,$$
$$S^{12}_{23} = S^{21}_{23} = 0.2251, S^{13}_{23} = S^{31}_{23} = 0.1971, S^{23}_{23} = S^{32}_{23} = 0.1034,$$
$$S^{12}_{24} = S^{21}_{24} = 0.0860, S^{13}_{24} = S^{31}_{24} = 0.0552, S^{23}_{24} = S^{23}_{24} = 0.1298,$$
$$S^{12}_{31} = S^{21}_{31} = 0.1789, S^{13}_{31} = S^{31}_{31} = 0.0778, S^{23}_{31} = S^{32}_{31} = 0.2237,$$
$$S^{12}_{32} = S^{21}_{32} = 0.1039, S^{13}_{32} = S^{31}_{32} = 0.1039, S^{23}_{32} = S^{23}_{32} = 0.1800,$$
$$S^{12}_{33} = S^{21}_{33} = 0.2968, S^{13}_{33} = S^{31}_{33} = 0.0954, S^{23}_{33} = S^{32}_{33} = 0.2895,$$
$$S^{12}_{34} = S^{21}_{34} = 0.1185, S^{13}_{34} = S^{31}_{34} = 0.1557, S^{23}_{34} = S^{23}_{34} = 0.0768,$$
$$S^{12}_{41} = S^{21}_{41} = 0.2022, S^{13}_{41} = S^{31}_{41} = 0.3730, S^{23}_{41} = S^{32}_{41} = 0.2247,$$
$$S^{12}_{42} = S^{21}_{42} = 0.2158, S^{13}_{42} = S^{31}_{42} = 0.1762, S^{23}_{42} = S^{23}_{42} = 0.3028,$$
$$S^{12}_{43} = S^{21}_{43} = 0.0962, S^{13}_{43} = S^{31}_{43} = 0.0917, S^{23}_{43} = S^{32}_{43} = 0.1856,$$
$$S^{12}_{44} = S^{21}_{44} = 0.1557, S^{13}_{44} = S^{31}_{44} = 0.1012, S^{23}_{44} = S^{23}_{44} = 0.0854,$$
$$S^{12}_{51} = S^{21}_{51} = 0.1775, S^{13}_{51} = S^{31}_{51} = 0.1801, S^{23}_{51} = S^{32}_{51} = 0.1617,$$
$$S^{12}_{52} = S^{21}_{52} = 0.0854, S^{13}_{52} = S^{31}_{52} = 0.1825, S^{23}_{52} = S^{23}_{52} = 0.1253,$$
$$S^{12}_{53} = S^{21}_{53} = 0.1851, S^{13}_{53} = S^{31}_{53} = 0.1990, S^{23}_{53} = S^{32}_{53} = 0.0604,$$
$$S^{12}_{54} = S^{21}_{54} = 0.0387, S^{13}_{54} = S^{31}_{54} = 0.0640, S^{23}_{54} = S^{23}_{54} = 0.0574.$$

Step 3. Determine the weighted support $T\left(\overline{\overline{sp}}^{f}_{hp}\right)$ by utilizing formula (64). For simplicity, we shall denote $T\left(\overline{\overline{sp}}^{f}_{hp}\right)$ by $T^{f}_{hp}(h = 1,\ldots,5;\ p = 1,\ldots,4, f = 1,\ldots,3)$.

$$T^{1}_{11} = 0.4241, T^{2}_{11} = 0.5735, T^{3}_{11} = 0.5254, T^{1}_{21} = 0.4791, T^{2}_{21} = 0.3034, T^{3}_{21} = 0.3596, T^{1}_{31} = 0.2567, T^{2}_{31} = 0.4026, T^{3}_{31} = 0.3015,$$
$$T^{1}_{41} = 0.5752, T^{2}_{41} = 0.4270, T^{3}_{41} = 0.5977, T^{1}_{51} = 0.3576, T^{2}_{51} = 0.3392, T^{3}_{51} = 0.3418, T^{1}_{12} = 0.4580, T^{2}_{12} = 0.3659, T^{3}_{12} = 0.3763,$$
$$T^{1}_{22} = 0.3645, T^{2}_{22} = 0.3188, T^{3}_{22} = 0.2462, T^{1}_{32} = 0.2079, T^{2}_{32} = 0.2839, T^{3}_{32} = 0.2839, T^{1}_{42} = 0.3920, T^{2}_{42} = 0.5186, T^{3}_{42} = 0.4790,$$
$$T^{1}_{52} = 0.2679, T^{2}_{52} = 0.2107, T^{3}_{52} = 0.3078, T^{1}_{13} = 0.2789, T^{2}_{13} = 0.3384, T^{3}_{13} = 0.2461, T^{1}_{23} = 0.4222, T^{2}_{23} = 0.3285, T^{3}_{23} = 0.3006,$$
$$T^{1}_{33} = 0.3922, T^{2}_{33} = 0.5863, T^{3}_{33} = 0.3849, T^{1}_{43} = 0.1878, T^{2}_{43} = 0.2818, T^{3}_{43} = 0.2773, T^{1}_{53} = 0.3841, T^{2}_{53} = 0.2455, T^{3}_{53} = 0.2594,$$
$$T^{1}_{14} = 0.3270, T^{2}_{14} = 0.4224, T^{3}_{14} = 0.4027, T^{1}_{24} = 0.1413, T^{2}_{24} = 0.2158, T^{3}_{24} = 0.1850, T^{1}_{34} = 0.2743, T^{2}_{34} = 0.1953, T^{3}_{34} = 0.2325,$$
$$T^{1}_{44} = 0.2570, T^{2}_{44} = 0.2412, T^{3}_{44} = 0.1867, T^{1}_{54} = 0.1028, T^{2}_{54} = 0.0962, T^{3}_{54} = 0.1215.$$

Step 4. Determine the weight Ψ^{f}_{hp} associated with SPFN $\overline{\overline{sp}}_{hp}(h = 1,\ldots,5;\ p = 1,\ldots,4, f = 1,2,3)$, by utilizing formula (65).

Table 10 Overall spherical fuzzy decision matrix utilizing SPDPWG

	$\overline{\overline{AS}}_1$	$\overline{\overline{AS}}_2$	$\overline{\overline{AS}}_3$	$\overline{\overline{AS}}_4$
$\overline{\overline{AV}}_1$	$\langle 0.6944, 0.2604, 0.4701\rangle$	$\langle 0.6939, 0.3452, 0.3406\rangle$	$\langle 0.7239, 0.3865, 0.2872\rangle$	$\langle 0.7496, 0.3127, 0.4634\rangle$
$\overline{\overline{AV}}_2$	$\langle 0.6936, 0.1742, 0.3122\rangle$	$\langle 0.6643, 0.2818, 0.3400\rangle$	$\langle 0.7251, 0.2438, 0.3272\rangle$	$\langle 0.7981, 0.2116, 0.2436\rangle$
$\overline{\overline{AV}}_3$	$\langle 0.7352, 0.1757, 0.3927\rangle$	$\langle 0.7120, 0.3164, 0.2170\rangle$	$\langle 0.6340, 0.3469, 0.3618\rangle$	$\langle 0.5791, 0.1880, 0.3976\rangle$
$\overline{\overline{AV}}_4$	$\langle 0.6441, 0.4170, 0.3647\rangle$	$\langle 0.4879, 0.4042, 0.4844\rangle$	$\langle 0.5887, 0.4648, 0.4407\rangle$	$\langle 0.6135, 0.3200, 0.4207\rangle$
$\overline{\overline{AV}}_5$	$\langle 0.4933, 0.3316, 0.3916\rangle$	$\langle 0.3743, 0.4343, 0.4627\rangle$	$\langle 0.4931, 0.4137, 0.4367\rangle$	$\langle 0.3433, 0.4634, 0.4850\rangle$

$\Psi_{11}^1 = 0.2960, \Psi_{11}^2 = 0.3698, \Psi_{11}^3 = 0.3342, \Psi_{21}^1 = 0.3372,$
$\Psi_{21}^2 = 0.3360, \Psi_{21}^3 = 0.3268, \Psi_{31}^1 = 0.2982, \Psi_{31}^2 = 0.3763,$
$\Psi_{31}^3 = 0.3255,$
$\Psi_{41}^1 = 0.3233, \Psi_{41}^2 = 0.3311, \Psi_{41}^3 = 0.3456, \Psi_{51}^1 = 0.3167,$
$\Psi_{51}^2 = 0.3532, \Psi_{51}^3 = 0.3300, \Psi_{12}^1 = 0.3274, \Psi_{12}^2 = 0.3468,$
$\Psi_{12}^3 = 0.3258,$
$\Psi_{22}^1 = 0.3273, \Psi_{22}^2 = 0.3576, \Psi_{22}^3 = 0.3151, \Psi_{32}^1 = 0.3010,$
$\Psi_{32}^2 = 0.3617, \Psi_{32}^3 = 0.3373, \Psi_{42}^1 = 0.2982, \Psi_{42}^2 = 0.3678,$
$\Psi_{42}^3 = 0.3340,$
$\Psi_{52}^1 = 0.3158, \Psi_{52}^2 = 0.34090, \Psi_{52}^3 = 0.3433, \Psi_{13}^1 = 0.3115,$
$\Psi_{13}^2 = 0.3686, \Psi_{13}^3 = 0.3199, \Psi_{23}^1 = 0.3311, \Psi_{23}^2 = 0.3497,$
$\Psi_{23}^3 = 0.3192,$
$\Psi_{33}^1 = 0.2997, \Psi_{33}^2 = 0.3861, \Psi_{33}^3 = 0.3143, \Psi_{43}^1 = 0.2982,$
$\Psi_{43}^2 = 0.3638, \Psi_{43}^3 = 0.3380, \Psi_{53}^1 = 0.3360, \Psi_{53}^2 = 0.3418,$
$\Psi_{53}^3 = 0.3223,$
$\Psi_{14}^1 = 0.3006, \Psi_{14}^2 = 0.3644, \Psi_{14}^3 = 0.3350, \Psi_{24}^1 = 0.3031,$
$\Psi_{24}^2 = 0.3651, \Psi_{24}^3 = 0.3318, \Psi_{34}^1 = 0.3247, \Psi_{34}^2 = 0.3443,$
$\Psi_{34}^3 = 0.3310,$
$\Psi_{44}^1 = 0.3214, \Psi_{44}^2 = 0.3588, \Psi_{44}^3 = 0.3198, \Psi_{54}^1 = 0.3129,$
$\Psi_{54}^2 = 0.3517, \Psi_{54}^3 = 0.3354.$

Step 5. Determine the overall evaluation value of each alternative $\overline{\overline{sp}}_h (h = 1, \ldots, 5)$ by utilizing formula (66), or formula (67) (assume $\gamma = 2$). The overall evaluation values are given in Tables 9 and 10.

Step 6. Determine the supports $Sup\left(\overline{\overline{sp}}_{hp}, \overline{\overline{sp}}_{hl}\right)$ by utilizing formula (68). For computational simplicity, we shall denote the support by $S_{hp,hl}$ instead of $Sup\left(\overline{\overline{sp}}_{hp}, \overline{\overline{sp}}_{lp}\right)(h = 1, \ldots, 5, p, l = 1, \ldots, 4)$.

$S_{11,12} = S_{12,11} = 0.1604, S_{11,13} = S_{13,11} = 0.0973,$
$S_{11,14} = S_{14,11} = 0.0985, S_{12,13} = S_{13,12} = 0.1477,$
$S_{12,14} = S_{14,12} = 0.2398, S_{13,14} = S_{14,13} = 0.1138;$
$S_{21,22} = S_{22,21} = 0.1840, S_{21,23} = S_{23,21} = 0.0608,$
$S_{21,24} = S_{24,21} = 0.0846, S_{22,23} = S_{23,22} = 0.1353,$
$S_{22,24} = S_{24,22} = 0.1105, S_{23,24} = S_{23,24} = 0.0580,$
$S_{31,32} = S_{32,31} = 0.1362, S_{31,33} = S_{33,31} = 0.0544,$
$S_{31,34} = S_{34,31} = 0.1543, S_{32,33} = S_{33,32} = 0.1292,$
$S_{32,34} = S_{34,32} = 0.2740, S_{33,34} = S_{34,33} = 0.1919;$
$S_{41,42} = S_{42,41} = 0.1945, S_{41,43} = S_{43,41} = 0.2614,$
$S_{41,44} = S_{44,41} = 0.1867, S_{42,43} = S_{43,42} = 0.3136,$
$S_{42,44} = S_{44,42} = 0.2639, S_{43,44} = S_{44,43} = 0.1025;$
$S_{51,52} = S_{52,51} = 0.2402, S_{51,53} = S_{53,51} = 0.0936,$
$S_{51,54} = S_{54,51} = 0.2377, S_{52,53} = S_{53,52} = 0.2449,$
$S_{52,54} = S_{54,52} = 0.3230, S_{53,54} = S_{54,53} = 0.1771.$

Or

$S_{11,12} = S_{12,11} = 0.1095, S_{11,13} = S_{13,11} = 0.1585,$
$S_{11,14} = S_{14,11} = 0.0540, S_{12,13} = S_{13,12} = 0.0522,$
$S_{12,14} = S_{14,12} = 0.0981, S_{13,14} = S_{14,13} = 0.1363;$
$S_{21,22} = S_{22,21} = 0.0813, S_{21,23} = S_{23,21} = 0.0551,$
$S_{21,24} = S_{24,21} = 0.0922, S_{22,23} = S_{23,22} = 0.0515,$
$S_{22,24} = S_{24,22} = 0.1267, S_{23,24} = S_{23,24} = 0.0817,$
$S_{31,32} = S_{32,31} = 0.1600, S_{31,33} = S_{33,31} = 0.1423,$
$S_{31,34} = S_{34,31} = 0.1108, S_{32,33} = S_{33,32} = 0.1183,$
$S_{32,34} = S_{34,32} = 0.1827, S_{33,34} = S_{34,33} = 0.1215;$
$S_{41,42} = S_{42,41} = 0.1394, S_{41,43} = S_{43,41} = 0.0746,$
$S_{41,44} = S_{44,41} = 0.0821, S_{42,43} = S_{43,42} = 0.0887,$
$S_{42,44} = S_{44,42} = 0.1160, S_{43,44} = S_{44,43} = 0.1048;$
$S_{51,52} = S_{52,51} = 0.1220, S_{51,53} = S_{53,51} = 0.0662,$
$S_{51,54} = S_{54,51} = 0.1558, S_{52,53} = S_{53,52} = 0.0872,$
$S_{52,54} = S_{54,52} = 0.0339, S_{53,54} = S_{54,53} = 0.1167.$

Table 11 Effect of the parameter γ on decision results

Parameter value	Score values utilizing SPFDPWA operator	Score values utilizing SPFDPWG operator	Ranking order
$\gamma = 1$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8402$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.8618$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8462$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.7847$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.6842$.	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.5385$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.5685$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.5442$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.5241$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.5150$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$ $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$
$\gamma = 4$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8696$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.8906$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8691$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8544$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7407$.	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.4929$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.5294$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.4924$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.4887$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.4992$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$ $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_5 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_4$
$\gamma = 9$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8840$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.9031$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8814$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8800$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7659$.	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.4687$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.5049$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.4698$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.4624$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.4891$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$ $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_5 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4$
$\gamma = 15$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8898$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.9080$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8867$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8879$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7750$.	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.4578$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.4949$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.4599$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.4507$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.4818$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$ $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_5 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4$
$\gamma = 20$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8922$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.9100$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8889$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8907$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7785$.	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.4535$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.4907$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.4555$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.4458$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.4780$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_5$ $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_5 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4$
$\gamma = 30$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8946$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.9119$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8912$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8935$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7821$.	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.4490$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.4864$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.4507$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.4407$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.4735$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_5$ $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_5$

Step 7. Determine the weighted support $T(\overline{\overline{sp}}_{hp})$ by utilizing formula (69). For simplicity, we shall denote $T(\overline{\overline{sp}}_{hp})$ by $T_{hp}(h = 1, \ldots, 5;\ p = 1, \ldots, 4)$.

$$T_{11} = 0.3562, T_{12} = 0.5480, T_{13} = 0.3588, T_{14} = 0.4521,$$
$$T_{21} = 0.3294, T_{22} = 0.4299, T_{23} = 0.2541, T_{24} = 0.2531;$$
$$T_{31} = 0.3450, T_{32} = 0.5394, T_{33} = 0.3755, T_{34} = 0.6202,$$
$$T_{41} = 0.6426, T_{42} = 0.7720, T_{43} = 0.6776, T_{44} = 0.5531;$$
$$T_{51} = 0.5715, T_{52} = 0.8081, T_{53} = 0.5156, T_{54} = 0.7377.$$

Or

$$T_{11} = 0.3220, T_{12} = 0.2598, T_{13} = 0.3469, T_{14} = 0.2883,$$
$$T_{21} = 0.2286, T_{22} = 0.2594, T_{23} = 0.1883, T_{24} = 0.3007;$$
$$T_{31} = 0.4131, T_{32} = 0.4610, T_{33} = 0.3820, T_{34} = 0.4150,$$
$$T_{41} = 0.2962, T_{42} = 0.3441, T_{43} = 0.2681, T_{44} = 0.3030;$$
$$T_{51} = 0.3441, T_{52} = 0.2431, T_{53} = 0.2701, T_{54} = 0.3065.$$

Determine the weight Θ_{hp} associated with SPFN $\overline{\overline{sp}}_{hp}(h = 1, \ldots, 5;\ p = 1, \ldots, 4)$, by utilizing formula (70).

$$\Theta_{11} = 0.2440, \Theta_{12} = 0.2677, \Theta_{13} = 0.2321, \Theta_{14} = 0.2562,$$
$$\Theta_{21} = 0.2595, \Theta_{22} = 0.2682, \Theta_{23} = 0.2324, \Theta_{24} = 0.2399;$$
$$\Theta_{31} = 0.2352, \Theta_{32} = 0.2587, \Theta_{33} = 0.2283, \Theta_{34} = 0.2778,$$
$$\Theta_{41} = 0.2542, \Theta_{42} = 0.2636, \Theta_{43} = 0.2465, \Theta_{44} = 0.2357;$$
$$\Theta_{51} = 0.2436, \Theta_{52} = 0.2693, \Theta_{53} = 0.2230, \Theta_{54} = 0.2641.$$

or

$$\Theta_{11} = 0.2605, \Theta_{12} = 0.2386, \Theta_{13} = 0.2520, \Theta_{14} = 0.2489,$$
$$\Theta_{21} = 0.2537, \Theta_{22} = 0.2500, \Theta_{23} = 0.2330, \Theta_{24} = 0.2634;$$
$$\Theta_{31} = 0.2561, \Theta_{32} = 0.2545, \Theta_{33} = 0.2378, \Theta_{34} = 0.2515,$$
$$\Theta_{41} = 0.2557, \Theta_{42} = 0.2548, \Theta_{43} = 0.2375, \Theta_{44} = 0.2520;$$
$$\Theta_{51} = 0.2674, \Theta_{52} = 0.2377, \Theta_{53} = 0.2399, \Theta_{54} = 0.2549.$$

Step 8. Determine the overall evaluation value of each alternative $\overline{\overline{sp}}_h(h = 1, \ldots, 5)$ by utilizing formula (71) (assume $\gamma = 2$).

$$\overline{\overline{sp}}_1 = \langle 0.7816, 0.2188, 0.0010 \rangle, \overline{\overline{sp}}_2 = \langle 0.7916, 0.1558, 0.0081 \rangle,$$
$$\overline{\overline{sp}}_3 = \langle 0.7424, 0.16890, 0.0024 \rangle,$$
$$\overline{\overline{sp}}_4 = \langle 0.6723, 0.2047, 0.0112 \rangle, \overline{\overline{sp}}_5 = \langle 0.4951, 0.3286, 0.0338 \rangle.$$

Or by formula (72), we have

Table 12 Comparison with other approaches

Approach	Score values	Ranking order
SPFDWA operator (Xu 2011) $(\gamma = 2)$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.9133$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.9685$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.9356$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.7707$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.6874$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$
SPFDWG operator (Xu 2011) $(\gamma = 2)$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8554$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.8382$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.7504$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.7401$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.6388$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$
SPFPWA operator (Chen 2018)	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.7346$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.7749$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.7422$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.6461$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.5599$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$
SPFPWG operator (Peng and Dai 2017)	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.5883$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.6165$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.6084$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.5752$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.5546$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$
Proposed SPFDPWA $(\gamma = 2)$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8539$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.8759$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8570$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8188$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7109$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$
Proposed SPFDPWG $(\gamma = 2)$	$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.5169$, $\overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.5532$, $\overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.5191$, $\overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.5102$, $\overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.5069$.	$\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$

$\overline{\overline{sp}}_1 = \langle 0.3076, 0.3371, 0.4197\rangle, \overline{\overline{sp}}_2 = \langle 0.2099, 0.2382, 0.3120\rangle,$
$\overline{\overline{sp}}_3 = \langle 0.2097, 0.2881, 0.3643\rangle,$
$\overline{\overline{sp}}_4 = \langle 0.3804, 0.4131, 0.4365\rangle, \overline{\overline{sp}}_5 = \langle 0.3903, 0.4206, 0.4491\rangle.$

Step 9. Determine the score values $\overline{\overline{SE}}(\overline{\overline{sp}}_h)$ by utilizing Definition 5, we have

$$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.8539, \ \overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.8759, \overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.8570, \overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.8188, \overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.7109.$$

or

$$\overline{\overline{SE}}(\overline{\overline{sp}}_1) = 0.5169, \ \overline{\overline{SE}}(\overline{\overline{sp}}_2) = 0.5532, \overline{\overline{SE}}(\overline{\overline{sp}}_3) = 0.5191, \overline{\overline{SE}}(\overline{\overline{sp}}_4) = 0.5102, \overline{\overline{SE}}(\overline{\overline{sp}}_5) = 0.5069.$$

Therefore, according to their score values the ranking order of the alternatives $\overline{\overline{AV}}_h (h = 1, 2, 3, 4, 5)$ is $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$ and $\overline{\overline{AV}}_2 > \overline{\overline{AV}}_3 > \overline{\overline{AV}}_1 > \overline{\overline{AV}}_4 > \overline{\overline{AV}}_5$.

Hence, by utilizing both the aggregation operators, we get the same ranking orders, So, according to the ranking order, the best emerging technology enterprise is $\overline{\overline{AV}}_2$ and the worst one is $\overline{\overline{AV}}_5$.

Effect of the parameter γ on decision results

In this subsection, we will further discuss the effect of the parameter γ on the final ranking result of this example, and then we adopt the different values of the parameter γ to rank the alternatives. The results are shown in Table 11. From Table 11, we can see that for different values of the parameter γ the ranking orders are different. However, the best alternative is $\overline{\overline{AV}}_2$, while the worst one is $\overline{\overline{AV}}_4$ or $\overline{\overline{AV}}_5$. From Table 11, we can also notice that when the value of the parameter γ increases while utilizing the SPFDPWA operator, the score values also increase, and when utilizing the SPFDPWG operator, the score values decrease.

Comparison with other approaches

In this subsection, we compare our developed approach with those proposed by Ashraf et al. (2019a, b), Mahmood et al. (Mahmood et al. 2018) and discuss the advantages of the proposed approach, and results are shown in Table 12.

From Table 12, we can see that the ranking order obtained from the developed approach and that of Ashraf et al. (2019a, b), Mahmood et al. (2018) are the same. This shows the validity of the proposed approach.

The main proposed aggregation operators have some advantages over the existing aggregation operators. (1) The developed aggregation operators can remove the influence of awful data from the decision result, (2) we apply the existing Dombi aggregation operators (Ashraf et al. 2019b) to solve the above MAGDM problem, the values obtained are not SPFNs, while the aggregated value obtained utilizing the developed aggregation operators is an SPFN. Similar limitations exist in the spherical fuzzy aggregation operators proposed by Mahmood et al. (2018), (3) the developed method is based on Dombi t-norm and t-conorm, and it is more flexible by general parameters. So, the decision-maker may choose the value of the general parameter according to the actual need of the situation.

Conclusion

In this paper, we first signify the notion of SPFNs in view of its three membership grades to depict uncertainties of real-world problems. We then pointed out some limitation in some proposed Dombi operational laws of SPFNs with the help of examples which leads us to propose some improved Dombi operational laws. Some featured achievements of this paper are as follows:

1. We proved the shortcomings of the existing Dombi operational laws with examples.
2. We present some new improved Dombi operational laws.
3. Based on new operations, we proposed improved Dombi aggregation operators and basic features of aggregation functions are examined for the proposed work.
4. We support the proposed theory with examples to justify the proposed work.
5. We proposed a MAGDM algorithm based on improved Dombi aggregation operators.
6. A comprehensive numerical example is investigated to show the application of proposed Dombi aggregation operators.
7. The proposed work is very useful in decision-making problems so it can be used for the evaluation of human resources (Gündoğdu and Kahraman 2019c; Gündoğdu et al. 2019; Jin et al. 2019), tourism recourses (Gündoğdu and Kahraman 2019c) and such useful work can be extended to some other fuzzy frameworks more generalized than SPFNs such as T-spherical fuzzy sets (TSFS) (Mahmood et al. 2018, 2020; Ullah et al. 2020; Munir et al. 2020), interval-valued TSFS (Ullah et al. 2019) and complex TSFS (Ali et al. 2020).

Declarations

Conflict of interest The authors declare no conflict of interest.

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