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On fixed point results for α_*-ψ- dominated fuzzy contractive mappings with graph

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Abstract. The purpose of this article is to establish fixed point results for a pair of α_*-dominated fuzzy mappings fulfilling generalized locally new α_*-ψ-Ćirić type rational contractive conditions on a closed ball in complete dislocated metric spaces. Example and application are given to demonstrate the novelty of our results. Our results extend several comparable results in the existing literature.

Keywords: Fixed point, complete dislocated metric space, α_*-dominated mapping, α_*-ψ-Ćirić type rational contraction, graphic contraction, fuzzy mappings

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1. Introduction and preliminaries

Let S be a metric space and $H : S \longrightarrow S$ be a mapping. A point $w \in S$ is called a fixed point of S if $w = Sw$. A lot of fixed point results for contractive mappings defined on the entire space hold. It is possible that $H : S \longrightarrow S$ is not contractive on whole space while $H : Y \subseteq S \longrightarrow S$ is a contraction. Shoaib et al. [27], proved the result related with $\alpha_* - \psi$-Ćirić type multivalued mappings by intersection of an iterative sequence on closed ball with graph. Recently Rasham et al. [21], proved fixed point results for a pair of multivalued mappings on closed ball for new rational type contraction in dislocated metric spaces. Further fixed point results on closed ball can be observed in [4, 19, 20, 29, 30].

Many authors proved fixed point theorems in complete dislocated metric space. The idea of dislocated topologies has useful applications in the context of logic programming semantics [12]. Dislocated metric space [16] is a generalization of partial metric space [17], which has applications in computer sciences. Nadler [18], started the research of fixed point results for the multivalued contractive mappings in a complete metric space. Asl et al. [5] gave the idea of α_*-ψ contractive multifunctions, α_*-admissible mapping and got some fixed point conclusions for these multifunctions can be seen in [1, 6, 28, 27]. Recently, Senapati and Dey [22], introduced the concept of a pair of multi β_*-admissible mapping and established some common fixed point theorems for multivalued β_* -ψ-contractive mappings. Recently, Alofi et al. [2] introduced the concept of α-dominated multivalued mappings and established some fixed point results for such mappings on a closed ball in complete dislocated quasi b -metric spaces.

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Since the notion of fuzzy sets a lot of work has been done in this area [33] Zadeh. In the field of fixed point theory Weiss [32] and Butnariu [8] presented the notion of fuzzy mappings and proved many concerned results. Heilpern [11] proved a fixed point result for fuzzy mappings which was the further analogue of Nadler's multivalued result [18] in Hausdorff metric. Motivated by the Heilpern's results, the fixed point theory for fuzzy contraction using the Hausdorff metric has become more mature in different directions by various authors [23–25].

In this paper, we establish common fixed point of α -dominated mappings for new Ćirić type rational fuzzy contractions on a closed ball in complete dislocated metric spaces. Interesting new results in metric space and partial metric space can be obtained as corollaries of our theorems. An application is derived in the setting of an ordered dislocated metric space for multi $\preceq$-dominated mappings. Also some new fixed point results with graphic contractions on closed ball for multi graph dominated fuzzy mappings on dislocated metric space are established. Example is given to show the superiority of our result. Our results generalize several comparable results in the existing literature.We give the following concepts which will be used in the paper.

Definition 1.1. Let M be a nonempty set and let $d_l : M \times M \to [0, \infty)$ be a function, called a dislocated metric (or simply d_l-metric), if for any $c, g, z \in M$, the following conditions satisfy:

i) If $d_l(c, g) = 0$, then $c = g$;
ii) $d_l(c, g) = d_l(g, c)$;
iii) $d_l(c, g) \leq d_l(c, z) + d_l(z, g) - d_l(z, z)$.

The pair (M, d_l) is called a dislocated metric space.

It is clear that if $d_l(c, g) = 0$, then from (i), $c = g$. But if $c = g$, $d_l(c, g)$ may not be 0. For $c \in M$ and $\varepsilon > 0$, $\overline{B(c, \varepsilon)} = \{g \in M : d_l(c, g) \leq \varepsilon\}$ is a closed ball in (M, d_l).We use $D.L.$space instead by dislocated metric space.

Example 1.2. [4] If $M = R^+ \cup \{0\}$, then $d_l(c, g) = c + g$ defines a dislocated metric d_l on M.

Definition 1.3. [4] Let (M, d_l) be a $D.L.$ space.

(i) A sequence $\{c_n\}$ in (M, d_l) is called Cauchy sequence if given $\varepsilon > 0$, there corresponds $n_0 \in N$ such that for all $n, m \geq n_0$ we have $d_l(c_m, c_n) < \varepsilon$ or $\lim_{n,m\to\infty} d_l(c_n, c_m) = 0$.

(ii) A sequence $\{c_n\}$ dislocated-converges (for short d_l -converges) to c if $\lim_{n\to\infty} d_l(c_n, c) = 0$. In this case c is called a d_l-limit of $\{c_n\}$.

(iii) (M, d_l) is called complete if every Cauchy sequence in M converges to a point $c \in M$ such that $d_l(c, c) = 0$.

Definition 1.4. [27] Let K be a nonempty subset of $D.L.$ space M and let $c \in M$. An element $g_0 \in K$ is called a best approximation in K if

$$d_l(c, K) = d_l(c, g_0), \text{ where } d_l(c, K) = \inf_{g\in K} d_l(c, g).$$

If each $c \in M$ has at least one best approximation in K, then K is called a proximinal set.

We denote $CP(M)$ be the set of all closed proximinal subsets of M. Let Ψ denote the family of all nondecreasing functions $\psi : [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=1}^{+\infty} \psi^n(t) < +\infty$ for all $t > 0$, where ψ^n is the n^{th} iterate of ψ. if $\psi \in \Psi$, then $\psi(t) < t$ for all $t > 0$.

Definition 1.5. [22] Let $S, T : M \to P(M)$ be the closed valued mulifunctions and $\beta : M \times M \to [0, +\infty)$ be a function. We say that the pair (S, T) is $\beta_\star$-admissible if for all $c, g \in M$

$$\beta(c, g) \geq 1 \Rightarrow \beta_\star(Sc, Tg) \geq 1, \text{ and } \beta_\star(Tc, Sg) \geq 1,$$

where $\beta_\star(Tc, Sg) = \inf\{\beta(a, b) : a \in Tc, \quad b \in Sg\}$. When $S = T$, then we obtain the definition of α_*-admissible mapping given in [5].

Definition 1.6. Let (M, d_l) be a $D.M.$space, $S, T : M \to P(M)$ be multivalued mappings and $\alpha : M \times M \to [0, +\infty)$. Let $A \subseteq M$, we say that the S is α_*-dominated on A, whenever $\alpha_*(c, Sc) \geq 1$ for all $c \in A$, where $\alpha_*(c, Sc) = \inf\{\alpha(c, b) : b \in Sc\}$. In particular, if A is a closed ball, we say that S is semi α_*-dominated on A. If $A = M$, then we say that the S is α_*-dominated on M. If $S, T : M \to M$ be self mappings, then S is α-dominated on A, whenever $\alpha(c, Sc) \geq 1$ for all $c \in A$.

Definition 1.7. [27] The function $H_{d_l} : P(M) \times P(M) \to R^+$, defined by

$$H_{d_l}(A, B) = \max\{\sup_{a\in A} d_l(a, B), \ \sup_{b\in B} d_l(A, b)\}$$

is called dislocated Hausdorff metric on $P(M)$.

Lemma 1.8. [27] *Let (M, d_l) be a D.L.space and $(P(M), H_{d_l})$ is a dislocated Hausdorff metric space* *on $P(M)$. If for all $H, U \in P(M)$ and for each $h \in H$, let $u_h \in U$ satisfying $d_l(h, U) = d_l(h, u_h)$. Then $H_{d_l}(H, U) \geq d_l(h, u_h)$.*

Example 1.9. Let $M = \mathbb{R}$. Define the mapping $\alpha : M \times M \to [0, \infty)$ by

$$\alpha(c, g) = \left\{ \begin{array}{c} 1 \text{ if } c > g \\ \frac{1}{2} \text{ otherwise} \end{array} \right\}.$$

Define the multivalued mappings $S, T : M \to P(M)$ by

$$Sc = \{[c-4, c-3] \text{ if } c \in M\}$$

and

$$Tg = \{[g-2, g-1] \text{ if } g \in M\}.$$

Suppose $c = 3$ and $g = 2$. As $3 > 2$, then $\alpha(3, 2) \geq 1$. Now, $\alpha_\star(S3, T2) = \inf\{\alpha(a, b) : a \in S3, b \in T2\} = \frac{1}{2} \ngeq 1$, this means $\alpha_\star(S3, T2) < 1$, that is, the pair (S, T) is not $\alpha_\star$-admissible. Also, $\alpha_\star(S3, S2) \ngeq 1$ and $\alpha_\star(T3, T2) \ngeq 1$. This implies S and T are not $\alpha_\star$-admissible individually. As, $\alpha_\star(c, Sc) = \inf\{\alpha(c, b) : b \in Sc\} \geq 1$, for all $c \in M$. Hence S is $\alpha_\star$-dominated mapping. Similarly $\alpha_\star(g, Tg) = \inf\{\alpha(g, b) : b \in Tg\} \geq 1$. Hence it is clear that S and T are $\alpha_\star$-dominated but not $\alpha_\star$-admissible.

Definition 1.10. [23] A fuzzy set A is a function with domain M and values in [0, 1], $F(M)$ is the collection of all fuzzy sets in M. If A is a fuzzy set and $c \in M$, then the function values $A(c)$ is called the grade of membership of c in A. The β-level set of fuzzy set T, is denoted by $[A]_\beta$, and defined as:

$$[A]_\beta = \{c : A(c) \geq \beta\} \quad \text{where} \quad 0 < \beta \leq 1,$$

$$[A]_0 = \overline{\{c : A(c) > 0\}}.$$

Now we select by the family $F(M)$ of all fuzzy sets, a subfamily with stronger properties, i.e. the subfamily of the approximate quantities, denoted $W(M)$ and defined by:

Definition 1.11. [11] A fuzzy subset A of M is an approximate quantity iff its β-level set is a compact convex subset of M for each $\beta \in [0, 1]$ and $\sup\limits_{x \in M} A(x) = 1$.

At this point, we can introduce a notion of fuzzy mapping, i.e., a mapping with value in the family of approximate quantities.

Definition 1.12. [11] Let M be an arbitrary set and Y any metric linear space. A mapping $T : M \to W(Y)$ is called a fuzzy mapping.

Note that we can see a fuzzy mapping $T : M \to W(Y)$ as a fuzzy subset of $M \times Y$, $T : M \times Y \to [0, 1]$, in the sense that $T(c, y) = T(c)(y)$.

Definition 1.13. [23] A point $c \in M$ is called a fuzzy fixed point of a fuzzy mapping $T : M \to W(M)$ if there exists $0 < \beta \leq 1$ such that $c \in [Tc]_\beta$.

2. Main result

Let (M, d_l) be a $D.L.$space, $c_0 \in M$ and $S, T : M \to W(M)$ be two fuzzy mappings on M. Moreover let $\gamma, \beta : M \to [0, 1]$ be two real functions. Let $c_1 \in [Sc_0]_{\gamma(c_0)}$ be an element such that $d_l(c_0, [Sc_0]_{\gamma(c_0)}) = d_l(c_0, c_1)$. Let $c_2 \in [Tc_1]_{\beta(c_1)}$ be such that $d_l(c_1, [Tc_1]_{\beta(c_1)}) = d_l(c_1, c_2)$. Let $c_3 \in [Sc_2]_{\gamma(c_2)}$ be such that $d_l(c_2, [Sc_2]_{\gamma(c_2)}) = d_l(c_2, c_3)$. Continuing this process, we construct a sequence c_n of points in M such that $c_{2n+1} \in [Sc_{2n}]_{\gamma(c_{2n})}$ and $c_{2n+2} \in [Tc_{2n+1}]_{\beta(c_{2n+1})}$, where $n = 0, 1, 2, \ldots.$ Also $d_l(c_{2n}, [Sc_2]_{\gamma(c_2)}) = d_l(c_{2n}, c_{2n+1})$, $d_l(c_{2n+1}, [Tc_{2n+1}]_{\beta(c_{2n+1})}) = d_l(c_{2n+1}, c_{2n+2})$.
We denote this iterative sequence by $\{TS(c_n)\}$. We say that $\{TS(c_n)\}$ is a sequence in M generated by c_0.

Theorem 2.1. *Let (M, d_l) be a complete $D.L.$ space. Let $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $S, T : M \to W(M)$ be two fuzzy α_*-dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that, for some $\psi \in \Psi$, $\gamma(c), \beta(g) \in (0, 1]$ and, for a suitable $a > 0$,*

$$D_l(c, g) = \max \left\{ \begin{array}{c} d_l(c, g), \dfrac{d_l(c, [Tg]_{\beta(g)}) + d_l(g, [Sc]_{\gamma(c)})}{2}, \\ \dfrac{d_l\left(c, [Sc]_{\gamma(c)}\right).d_l\left(g, [Tg]_{\beta(g)}\right)}{a + d_l(c, g)}, d_l(c, [Sc]_{\gamma(c)}), d_l(g, [Tg]_{\beta(g)}) \end{array} \right\},$$

the following hold:

$$H_{d_l}([Sc]_{\gamma(c)}, [Tg]_{\beta(g)}) \leq \psi(D_l(c, g)), \tag{2.1}$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$, *with either* $\alpha(c, g) \geq 1$ *or* $\alpha(g, c) \geq 1$. *Furthermore, suppese that*

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, [Sc_0]_{\gamma(c_0)})) \leq r \text{ for all } n \in N \cup \{0\}. \tag{2.2}$$

Then $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \geq 1$ for all $n \in N \cup \{0\}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0, r)}$. Also if $\alpha(c_n, c^*) \geq 1$ or $\alpha(c^*, c_n) \geq 1$ for all $n \in N \cup \{0\}$ the inequality (2.1) holds for c^* also. Then S and T have common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0, r)}$.

Proof. Consider a sequence $\{TS(c_0)\}$. From (2.2), we get

$$d_l(c_0, c_1) \leq \sum_{i=0}^{n} \psi^i(d_l(c_0, c_1)) \leq r.$$

It follows that,

$$c_1 \in \overline{B_{d_l}(c_0, r)}.$$

Let $c_3, \cdots, c_j \in \overline{B_{d_l}(c_0, r)}$ for some $j \in N$. If $j = 2i + 1$, where $i = 0, 1, 2, \ldots, \frac{j-1}{2}$. Since $S, T : M \to P(M)$ be an α_*−dominated mappings on $\overline{B_{d_l}(c_0, r)}$, so $\alpha_*(c_{2i}, Sc_{2i}) \geq 1$ and $\alpha_*(c_{2i+1}, Tc_{2i+1}) \geq 1$. Now by using Lemma 1.8, we obtain,

$$\begin{aligned}
& d_l(c_{2i+1}, c_{2i+2}) \\
& \leq H_{d_l}([Sc_{2i}]_{\gamma(c_{2i})}, [Tc_{2i+1}]_{\beta(c_{2i+1})}) \\
& \leq \psi\left(\max\left\{\begin{array}{c} d_l(c_{2i}, c_{2i+1}), \dfrac{d_l\left(c_{2i}, [Tc_{2i+1}]_{\beta(c_{2i+1})}\right) + d_l\left(c_{2i+1}, [Sc_{2i}]_{\gamma(c_{2i})}\right)}{2}, \\ \dfrac{d_l\left(c_{2i}, [Sc_{2i}]_{\gamma(c_{2i})}\right) .d_l\left(c_{2i+1}, [Tc_{2i+1}]_{\beta(c_{2i+1})}\right)}{a + d_l\left(c_{2i}, c_{2i+1}\right)}, \\ d_l(c_{2i}, [Sc_{2i}]_{\gamma(c_{2i})}), d_l(c_{2i+1}, [Tc_{2i+1}]_{\beta(c_{2i+1})}) \end{array}\right\}\right) \\
& \leq \psi\left(\max\left\{\begin{array}{c} d_l(c_{2i}, c_{2i+1}), \dfrac{d_l\left(c_{2i}, c_{2i+2}\right) + d_l\left(c_{2i+1}, c_{2i+1}\right)}{2}, \\ \dfrac{d_l\left(c_{2i}, c_{2i+1}\right) .d_l\left(c_{2i+1}, c_{2i+2}\right)}{a + d_l\left(c_{2i}, c_{2i+1}\right)}, d_l(c_{2i}, c_{2i+1}), d_l(c_{2i+1}, c_{2i+2}) \end{array}\right\}\right)
\end{aligned}$$

$$\begin{aligned}
& d_l(c_{2i+1}, c_{2i+2}) \\
& \leq \psi\left(\max\left\{\begin{array}{c} d_l(c_{2i}, c_{2i+1}), \\ \dfrac{d_l\left(c_{2i}, c_{2i+1}\right) + d_l\left(c_{2i+1}, c_{2i+2}\right) - d_l\left(c_{2i+1}, c_{2i+1}\right) + d_l\left(c_{2i+1}, c_{2i+1}\right)}{2}, \\ \dfrac{d_l\left(c_{2i}, c_{2i+1}\right) .d_l\left(c_{2i+1}, c_{2i+2}\right)}{a + d_l\left(c_{2i}, c_{2i+1}\right)}, d_l(c_{2i}, c_{2i+1}), d_l(c_{2i+1}, c_{2i+2}) \end{array}\right\}\right) \\
& \leq \psi\left(\max\left\{\begin{array}{c} d_l(c_{2i}, c_{2i+1}), \dfrac{d_l\left(c_{2i}, c_{2i+1}\right) + d_l\left(c_{2i+1}, c_{2i+2}\right)}{2}, \\ \dfrac{d_l\left(c_{2i}, c_{2i+1}\right) .d_l\left(c_{2i+1}, c_{2i+2}\right)}{a + d_l\left(c_{2i}, c_{2i+1}\right)}, d_l(c_{2i}, c_{2i+1}), d_l(c_{2i+1}, c_{2i+2}) \end{array}\right\}\right) \\
& \leq \psi\left(\max\left\{d_l(c_{2i}, c_{2i+1}), d_l(c_{2i+1}, c_{2i+2})\right\}\right).
\end{aligned}$$

If $\max\{d_l(c_{2i}, c_{2i+1}), d_l(c_{2i+1}, c_{2i+2})\} = d_l(c_{2i+1}, c_{2i+2})$, then $d_l(c_{2i+1}, c_{2i+2}) \leq \psi(d_l(c_{2i+1}, c_{2i+2}))$. This is the contradiction to the fact that $\psi(t) < t$ for all $t > 0$. So $\max\{d_l(c_{2i}, c_{2i+1}), d_l(c_{2i+1}, c_{2i+2})\} = d_l(c_{2i}, c_{2i+1})$. Hence, we obtain

$$d_l(c_{2i+1}, c_{2i+2}) \leq \psi(d_l(c_{2i}, c_{2i+1})). \tag{2.3}$$

As $\alpha_*(c_{2i}, Sc_{2i}) \geq 1$ and $c_{2i+1} \in Sc_{2i}$, so $\alpha(c_{2i}, c_{2i+1}) \geq 1$. Similarly we can get $\alpha_*(c_{2i-1}, [Tc_{2i-1}]_{\beta(c_{2i-1})}) \geq 1$ and $c_{2i-1} \in [Tc_{2i-1}]_{\beta(c_{2i-1})}$, so $\alpha(c_{2i-1}, c_{2i}) \geq 1$. Now by using (2.1), and Lemma 1.8, we have

$$\begin{aligned}
d_l(c_{2i}, c_{2i+1}) &\leq H_{d_l}([Tc_{2i-1}]_{\beta(c_{2i-1})}, [Sc_{2i}]_{\gamma(c_{2i})}) = H_{d_l}([Sc_{2i}]_{\gamma(c_{2i})}, [Tc_{2i-1}]_{\beta(c_{2i-1})}) \\
&\leq \psi\left(\max\left\{\begin{array}{c} d_l(c_{2i}, c_{2i-1}), \\ \dfrac{d_l\left(c_{2i}, [Tc_{2i-1}]_{\beta(c_{2i-1})}\right) + d_l\left(c_{2i-1}, [Sc_{2i}]_{\gamma(c_{2i})}\right)}{2}, \\ \dfrac{d_l\left(c_{2i}, [Sc_{2i}]_{\gamma(c_{2i})}\right) .d_l\left(c_{2i-1}, [Tc_{2i-1}]_{\beta(c_{2i-1})}\right)}{a + d_l\left(c_{2i}, c_{2i-1}\right)}, \\ d_l(c_{2i}, [Sc_{2i}]_{\gamma(c_{2i})}), d_l(c_{2i-1}, [Tc_{2i-1}]_{\beta(c_{2i-1})}) \end{array}\right\}\right) \\
&\leq \psi\left(\max\left\{\begin{array}{c} d_l(c_{2i}, c_{2i-1}), \dfrac{d_l\left(c_{2i}, c_{2i}\right) + d_l\left(c_{2i-1}, c_{2i+1}\right)}{2}, \\ \dfrac{d_l\left(c_{2i}, c_{2i+1}\right) .d_l\left(c_{2i-1}, c_{2i}\right)}{a + d_l\left(c_{2i}, c_{2i-1}\right)}, d_l(c_{2i}, c_{2i+1}), d_l(c_{2i-1}, c_{2i}) \end{array}\right\}\right) \\
&\leq \psi(\max\{d_l(c_{2i}, c_{2i-1}), d_l(c_{2i}, c_{2i+1}), d_l(c_{2i-1}, c_{2i})\}).
\end{aligned}$$

If $\max\{d_l(c_{2i}, c_{2i-1}), d_l(c_{2i}, c_{2i+1})\} = d_l(c_{2i}, c_{2i+1})$, then

$$d_l(c_{2i}, c_{2i+1}) \leq \psi(d_l(c_{2i}, c_{2i+1})).$$

This is the contradiction to the fact that $\psi(t) < t$ for all $t > 0$. If

$$\max\{d_l(c_{2i}, c_{2i-1}), d_l(c_{2i-1}, c_{2i})\} = d_l(c_{2i-1}, c_{2i}),$$

then

$$d_l(c_{2i}, c_{2i+1})) \leq \psi(d_l(c_{2i-1}, c_{2i})).$$

As ψ is nondecreasing function, so

$$\psi(d_l(c_{2i}, c_{2i+1})) \leq \psi^2(d_l(c_{2i-1}, c_{2i})),$$

by using the above inequality in (2.3), we obtain

$$d_l(c_{2i+1}, c_{2i+2}) \leq \psi^2(d_l(c_{2i-1}, c_{2i})),$$

continuing in this way, we obtain

$$d_l(c_{2i+1}, c_{2i+2}) \leq \psi^{2i+1}(d_l(c_0, c_1)). \tag{2.4}$$

Now, if $j = 2i$, where $i = 1, 2, \ldots \frac{j}{2}$. Then, similarly, we have

$$d_l(c_{2i}, c_{2i+1}) \leq \psi^{2i}(d_l(c_1, c_0)). \tag{2.5}$$

Now, by combining (2.4) and (2.5), we obtain

$$d_l(c_j, c_{j+1}) \leq \psi^j(d_l(c_1, c_0)) \text{ for some } j \in N. \tag{2.6}$$

Now,

$$\begin{aligned}
d_l(c_0, c_{j+1}) &\leq d_l(c_0, c_1) + \ldots + d_l(c_j, c_{j+1}) \\
&\leq d_l(c_0, c_1) + \ldots + \psi^j(d_l(c_0, c_1)), \quad \text{by (2.6)} \\
d_l(c_0, c_{j+1}) &\leq \sum_{i=0}^{j} \psi^i(d_l(c_0, c_1)) \leq r. \quad \text{by (2.2)}
\end{aligned}$$

Thus $c_{j+1} \in \overline{B_{d_l}(c_0, r)}$. Hence $c_n \in \overline{B_{d_l}(c_0, r)}$ for all $n \in N$ therefore $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$. As $S, T : M \to W(M)$ be two fuzzy semi α_*-dominated mappings on $\overline{B_{d_l}(c_0, r)}$, so $\alpha_*(c_n, [Sc_n]_{\gamma(c_n)}) \geq 1$ and $\alpha_*(c_n, [Tc_n]_{\beta(c_n)}) \geq 1$, for all $n \in N$. Now inequality (2.6) can be written as

$$d_l(c_n, c_{n+1}) \leq \psi^n(d_l(c_0, c_1)), \text{ for all } n \in N. \tag{2.7}$$

Fix $\varepsilon > 0$ and let $n(\varepsilon) \in N$ such that $\sum_{k \geq n(\varepsilon)} \psi^k (d_l(c_0, c_1)) < \varepsilon$. Let $n, m \in N$ with $m > n > n(\varepsilon)$, then, we obtain,

$$\begin{aligned}
d_l(c_n, c_m) &\leq \sum_{i=n}^{m-1} d_l(c_i, c_{i+1}) \leq \sum_{i=n}^{m-1} \psi^i(d_l(c_0, c_1)) \\
&\leq \sum_{k \geq n(\varepsilon)} \psi^k(d_l(c_0, c_1)) < \varepsilon.
\end{aligned}$$

Thus we proved that $\{TS(c_0)\}$ is a Cauchy sequence in $(\overline{B_{d_l}(c_0, r)}, d_l)$. As every closed ball in a complete $D.L.$ space is complete, so there exists $c^* \in \overline{B_{d_l}(c_0, r)}$ such that $\{TS(c_n)\} \to c^*$, that is

$$\lim_{n\to\infty} d_l(c^*, c_n) = 0. \tag{2.8}$$

By assumption $\alpha(c^*, c_{2n+1}) \geq 1$ for all $n \in N \cup \{0\}$. Since S and T are α_*-dominated, by Definition 1.6 we have that $\alpha_*(c^*, [Sc^*]_{\gamma(c^*)}) \geq 1$ and $\alpha_*(c_{2n+1}, [Tc_{2n+1}]_{\beta(c_{2n+1})}) \geq 1$. Now by using Lemma 1.8 and inequality (2.1), we have

$d_l(g, [Tg]_{\beta(g)})\}$, *the following hold:*

$$H_{d_l}([Sc]_{\gamma(c)}, [Tg]_{\beta(g)})\} \leq \psi(D_{l'}(c, g)), \tag{2.9}$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$ *with either* $\alpha(c, g) \geq 1$ *or* $\alpha(g, c) \geq 1$. *Also*

$$\begin{aligned}
d_l(c^*, [Sc^*]_{\gamma(c^*)}) &\leq d_l(c^*, c_{2n+2}) + d_l(c_{2n+2}, [Sc^*]_{\gamma(c^*)}) \\
&\leq d_l(c^*, c_{2n+2}) + H_{d_l}([Tc_{2n+1}]_{\beta(c_{2n+1})}, [Sc^*]_{\gamma(c^*)}) \\
&\leq d_l(c^*, c_{2n+2}) + H_{d_l}([Sc^*]_{\gamma(c^*)}, [Tc_{2n+1}]_{\beta(c_{2n+1})}) \\
&\leq d_l(c^*, c_{2n+2}) + \psi(D_l(c^*, c_{2n+1})) \\
&\leq d_l(c^*, c_{2n+2}) \\
&\quad + \psi\left(\max\left\{\begin{array}{c} d_l(c^*, c_{2n+1}), \\ \dfrac{d_l\left(c^*, c_{2n+2}\right) + d_l\left(c_{2n+1}, [Sc^*]_{\gamma(c^*)}\right)}{2}, \\ \dfrac{d_l\left(c^*, [Sc^*]_{\gamma(c^*)}\right).d_l\left(c_{2n+1}, c_{2n+2}\right)}{a + d_l\left(c^*, c_{2n+1}\right)}, \\ d_l(c^*, [Sc^*]_{\gamma(c^*)}), d_l(c_{2n+1}, c_{2n+2}) \end{array}\right\}\right).
\end{aligned}$$

Letting $n \to \infty$, and using the inequalities (2.7) and (2.8), we can easily get that $d_l(c^*, [Sc^*]_{\gamma(c^*)}) \leq \psi(d_l(c^*, [Sc^*]_{\gamma(c^*)}))$ and hence $d_l(c^*, [Sc^*]_{\gamma(c^*)}) \leq 0$ or $c^* \in [Sc^*]_{\gamma(c^*)}$. Similarly, by using,

$$\begin{aligned}
&d_l(c^*, [Tc^*]_{\beta(c^*)}) \\
&\leq d_l(c^*, c_{2n+1}) + d_l(c_{2n+1}, [Tc^*]_{\beta(c^*)}),
\end{aligned}$$

we can show that $c^* \in Tc^*$. Hence S and T have a common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0, r)}$. Since $\alpha_*(c^*, [Sc^*]_{\gamma(c^*)}) \geq 1$ and (S, T) be the pair of sub α_*-dominated multifunction on $\overline{B_{d_l}(c_0, r)}$, we have $\alpha_*(c^*, [Tc^*]_{\beta(c^*)}) \geq 1$ so $\alpha(c^*, c^*) \geq 1$. Now,

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, c_1)) \leq r \textit{ for all } n \in N \cup \{0\}.$$

Then $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0, r)}$. Also, if the inequality (2.9) holds for c^* and either $\alpha(c_n, c^*) \geq 1$ or $\alpha(c^*, c_n) \geq 1$ for all $n \in N \cup \{0\}$. Then S and T have a common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0, r)}$ and $d_l(c^*, c^*) = 0$.

$$\begin{aligned}
d_l(c^*, c^*) &\leq d_l(c^*, [Tc^*]_{\gamma(c^*)}) \leq H_{d_l}([Sc^*]_{\gamma(c^*)}, [Tc^*]_{\beta(c^*)}) \\
&\leq \psi\left(\max\left\{\begin{array}{c} d_l(c^*, c^*), \dfrac{d_l\left(c^*, [Tc^*]_{\beta(c^*)}\right) + d_l\left(c^*, [Sc^*]_{\gamma(c^*)}\right)}{2}, \\ \dfrac{d_l\left(c^*, [Sc^*]_{\gamma(c^*)}\right).d_l\left(c^*, [Tc^*]_{\beta(c^*)}\right)}{a + d_l\left(c^*, c^*\right)}, \\ d_l(c^*, [Sc^*]_{\gamma(c^*)}), d_l(c^*, [Tc^*]_{\beta(c^*)}) \end{array}\right\}\right).
\end{aligned}$$

This implies that, $d_l(c^*, c^*) = 0$.

Corollary 2.2. *Let* (M, d_l) *be a complete D.L. space. Take a function* $\alpha : M \times M \to [0, \infty)$. *Let,* $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ *and* $S, T : M \to W(M)$ *be two fuzzy semi* α_*-*dominated mappings on* $\overline{B_{d_l}(c_0, r)}$. *Assume that, for some* $\psi \in \Psi$, $\gamma(c)$, $\beta(g) \in (0, 1]$ *and* $D_{l'}(c, g) = \max\{d_l(c, g), d_l(c, [Sc]_{\gamma(c)}),$

Corollary 2.3. *Let* (M, d_l) *be a complete D.L. space. Take a function* $\alpha : M \times M \to [0, \infty)$. *Let,* $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ *and* $S : M \to W(M)$ *be a semi* α_*-*dominated fuzzy mappings on* $\overline{B_{d_l}(c_0, r)}$. *Assume that, for some* $\psi \in \Psi$, $\gamma(g) \in (0, 1]$ *and, for a suitable* $a > 0$,

$$D_{l''}(c,g) = \max\left\{\begin{array}{c} d_l(c,g), \dfrac{d_l\left(c,[Sg]_{\gamma(g)}\right)+d_l\left(g,[Sc]_{\gamma(c)}\right)}{2}, \\ \dfrac{d_l\left(c,[Sc]_{\gamma(c)}\right).d_l\left(g,[Sg]_{\gamma(g)}\right)}{a+d_l(c,g)}, \\ d_l(c,[Sc]_{\gamma(c)}), d_l(g,[Sg]_{\gamma(g)}) \end{array}\right\}$$

the following hold:

$$H_{d_l}([Sc]_{\gamma(c)},[Sg]_{\gamma(g)}) \leq \psi(D_{l''}(c,g)), \qquad (2.10)$$

for all $c, g \in \overline{B_{d_l}(c_0,r)} \cap \{S(c_n)\}$ *with* $\alpha(c,g) \geq 1$. *Also*

$$\sum_{i=0}^{n} \psi^i(d_l(c_0,c_1)) \leq r \text{ for all } n \in N \cup \{0\}.$$

Then $\{S(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$ and $\{S(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. Also, if the inequality (2.10) holds for c^* and either $\alpha(c_n,c^*) \geq 1$ or $\alpha(c^*,c_n) \geq 1$ for all $n \in N \cup \{0\}$. Then S has a fuzzy fixed point c^* in $\overline{B_{d_l}(c_0,r)}$ and $d_l(c^*,c^*) = 0$.

Let M be a nonempty set, $\preceq$ is a partial order on M and $A \subseteq M$. We say that $a \preceq B$ whenever for all $b \in B$, we have $a \preceq b$. A mapping $S : M \to W(M)$ is said to be prevalent on A if $a \preceq Sa$ for each $a \in A \subseteq M$. If $A = M$, then $S : M \to W(M)$ is said to be totally prevalent.

Theorem 2.4. *Let* $(M, \preceq, d_l)$ *be an ordered complete D.L. space. Let,* $r > 0$, $c_0 \in \overline{B_{d_l}(c_0,r)}$ *and* $S, T : M \to W(M)$ *be two fuzzy prevalent mappings on* $\overline{B_{d_l}(c_0,r)}$. *Assume that, for some* $\psi \in \Psi$, $\gamma(c), \beta(g) \in (0,1]$ *and, for a suitable* $a > 0$,

$$D_l(c,g) = \max\left\{\begin{array}{c} d_l(c,g), \dfrac{d_l\left(c,[Tg]_{\beta(g)}\right)+d_l\left(g,[Sc]_{\gamma(c)}\right)}{2}, \\ \dfrac{d_l\left(c,[Sc]_{\gamma(c)}\right).d_l\left(g,[Tg]_{\beta(g)}\right)}{a+d_l(c,g)}, \\ d_l(c,[Sc]_{\gamma(c)}), d_l(g,[Tg]_{\beta(g)}) \end{array}\right\}$$

the following hold:

$$H_{d_l}([Sc]_{\gamma(c)},[Tg]_{\beta(g)}) \leq \psi(D_l(c,g)) \qquad (2.11)$$

for all $c, g \in \overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}$ *with either* $c \preceq g$ *or* $g \preceq c$. *Also*

$$\sum_{i=0}^{n} \psi^i(d_l(c_1,c_0)) \leq r \text{ for all } n \in N \cup \{0\}. \qquad (2.12)$$

Then $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. Also if the inequality (2.11) holds for c^* and either $c_n \preceq c^*$ or $c^* \preceq c_n$ for all $n \in N \cup \{0\}$. Then S and T have a common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0,r)}$ and $d_l(c^*,c^*) = 0$.

Proof. Let $\alpha : M \times M \to [0,+\infty)$ be a mapping defined by $\alpha(c,g) = 1$ for all $c \in \overline{B_{d_l}(c_0,r)}$ with $c \preceq g$, and $\alpha(c,g) = 0$ for all other elements $c, g \in M$. As S and T are the fuzzy prevalent mappings on $\overline{B_{d_l}(c_0,r)}$, so $c \preceq [Sc]_{\gamma(c)}$ and $c \preceq [Tc]_{\beta(c)}$ for all $c \in \overline{B_{d_l}(c_0,r)}$. This implies that $c \preceq b$ for all $b \in [Sc]_{\gamma(c)}$ and $c \preceq e$ for all $c \in [Tc]_{\beta(c)}$. So, $\alpha(c,b) = 1$ for all $b \in [Sc]_{\gamma(c)}$ and $\alpha(c,e) = 1$ for all $c \in [Tc]_{\beta(c)}$. This implies that $\inf\{\alpha(c,g) : g \in [Sc]_{\gamma(c)}\} = 1$, and $\inf\{\alpha(c,g) : g \in [Tc]_{\beta(c)}\} = 1$. Hence $\alpha_*(c,[Sc]_{\alpha(c)}) = 1$, $\alpha_*(c,[Tc]_{\beta(c)}) = 1$ for all $c \in \overline{B_{d_l}(c_0,r)}$. So, $S, T : M \to W(M)$ are the semi α_*−dominated mapping on $\overline{B_{d_l}(c_0,r)}$. Moreover, inequality (2.11) can be written as

$$H_{d_l}([Sc]_{\gamma(c)},[Tg]_{\beta(g)}) \leq \psi(D_l(c,g)),$$

for all elements c, g in $\overline{B_{d_l}(c_0,r)} \cap \{TS(c_n)\}$, with either $\alpha(c,g) \geq 1$ or $\alpha(g,c) \geq 1$. Also, inequality (2.12) holds. Then, by Theorem 2.1, we have $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0,r)}$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0,r)}$. Now, $c_n, c^* \in \overline{B_{d_l}(c_0,r)}$ and either $c_n \preceq c^*$ or $c^* \preceq c_n$ implies that either $\alpha(c_n,c^*) \geq 1$ or $\alpha(c^*,c_n) \geq 1$. So, all the conditions of Theorem 2.1 are satisfied. Hence, by Theorem 2.1, S and T have a common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0,r)}$ and $d_l(c^*,c^*) = 0$.

Example 2.5. Let $M = Q^+ \cup \{0\}$ and let $d_l : M \times M \to M$ be the complete dislocated metric on M defined by

$$d_l(c,g) = c + g \text{ for all } c, g \in M.$$

Define the fuzzy mappings, $S, T : M \to W(M)$ by,

$$(Sc)(t) = \begin{cases} \gamma & \text{if } \frac{x}{4} \leq t < \frac{x}{2} \\ \frac{\gamma}{2} & \text{if } \frac{x}{2} \leq t \leq \frac{3x}{4} \\ \frac{\gamma}{4} & \text{if } \frac{3x}{4} < t \leq x \\ 0 & \text{if } x < t < \infty \end{cases}$$

and,

$$(Tg)(t) = \begin{cases} \beta & \text{if } \frac{x}{3} \leq t < \frac{x}{2} \\ \frac{\beta}{4} & \text{if } \frac{x}{2} \leq t \leq \frac{2x}{3} \\ \frac{\beta}{6} & \text{if } \frac{2x}{3} < t \leq x \\ 0 & \text{if } x < t < \infty \end{cases}.$$

Now, we consider

$$[Sc]_{\frac{\gamma}{2}} = \left[\frac{x}{4}, \frac{3x}{4}\right] \text{ and } \left[Tg\right]_{\frac{\beta}{4}} = \left[\frac{x}{3}, \frac{2x}{3}\right]$$

Considering, $c_0 = \frac{1}{2}$, $r = 5$, then $\overline{B_{d_l}(c_0, r)} = [0, \frac{9}{2}] \cap M$. Now we have $d_l(c_0, Sc_0) = d_l(\frac{1}{2}, S\frac{1}{2}) = d_l(\frac{1}{2}, \frac{1}{8}) = \frac{5}{8}$. So we obtain a sequence $\{TS(c_n)\} = \{\frac{1}{2}, \frac{1}{8}, \frac{1}{24}, \frac{1}{96}, ...\}$ in M generated by c_0.Let $\psi(t) = \frac{3t}{4}$, $a = \frac{1}{2}$ and,

$$\alpha(c, g) = \begin{cases} 1 & \text{if } c, g \in [0, \frac{9}{2}] \\ \frac{5}{4} & \text{otherwise.} \end{cases}$$

Now take $5, 6 \in M$, then, we have

$$H_{d_l}([S5]_{\frac{\gamma}{2}}, [T6]_{\frac{\beta}{4}}) = \frac{23}{4} > \psi(D_l(M, g)) = \frac{33}{4}.$$

So, the contractive condition does not hold on whole space M. Now for all $c, g \in \overline{B_{d_q}(c_0, r)} \cap \{TS(c_n)\}$ with either $\alpha(c, g) \geq 1$ or $\alpha(g, c) \geq 1$, we have

$$\begin{aligned}
&H_{d_l}([Sc]_{\frac{\gamma}{2}}, \left[Tg\right]_{\frac{\beta}{4}}) \\
&= \max\left\{\sup_{a\in[Sc]_{\frac{\gamma}{2}}} d_l\left(a, \left[Tg\right]_{\frac{\beta}{4}}\right), \sup_{b\in\left[Tg\right]_{\frac{\beta}{4}}} d_l\left([Sc]_{\frac{\gamma}{2}}, b\right)\right\} \\
&= \max\left\{\sup_{a\in Sc} d_l\left(a, \left[\frac{g}{3}, \frac{2g}{3}\right]\right), \sup_{b\in Tg} d_l\left(\left[\frac{c}{4}, \frac{3c}{4}\right], b\right)\right\} \\
&= \max\left\{d_l\left(\frac{3c}{4}, \left[\frac{g}{3}, \frac{2g}{3}\right]\right), d_l\left(\left[\frac{c}{4}, \frac{3c}{4}\right], \frac{2g}{3}\right)\right\} \\
&= \max\left\{d_l\left(\frac{3c}{4}, \frac{g}{3}\right), d_l\left(\frac{c}{4}, \frac{2g}{3}\right)\right\} \\
&= \max\left\{\frac{3c}{4} + \frac{g}{3}, \frac{c}{4} + \frac{2g}{3}\right\} \\
&\leq \psi\left(\max\left\{c + g, \frac{10cg}{1 + 2c + 2g}, \frac{15c + 16g}{24}, \frac{4g}{3}, \frac{5c}{4}\right\}\right) \\
&= \psi(D_l(c, g)).
\end{aligned}$$

So, the contractive condition holds on $\overline{B_{d_q}(c_0, r)} \cap \{TS(c_n)\}$. Also,

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, c_1)) = \frac{5}{8}\sum_{i=0}^{n}(\frac{3}{4})^i < 5 = r.$$

Hence, all the conditions of Theorem 2.1 are satisfied. Now, we have $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \geq 1$ and $\{TS(c_n)\} \to 0 \in \overline{B_{d_l}(c_0, r)}$. Also, $\alpha(c_n, 0) \geq 1$ or $\alpha(0, c_n) \geq 1$ for all $n \in N \cup \{0\}$. Moreover, 0 is a common fuzzy fixed point of S and T.

3. Fixed point results for graphic contractions

In this section we presents an application of Theorem 2.1 in graph theory. Jachymski, [15], proved the result concerning for contraction mappings on metric space with a graph. Hussain et al. [14], introduced the fixed points theorem for graphic contraction and gave an application. A graph K is connected if there is a path between any two different vertices (see for detail [7, 31]).

Definition 3.1. Let M be a nonempty set and $K = (V(K), F(K))$ be a graph such that $V(K) = M$, $A \subseteq M$. A mapping $S : M \to P(M)$ is said to be multi graph dominated on A if $(c, g) \in F(K)$, for all $g \in Sc$ and $c \in A$.

Theorem 3.2. *Let (M, d_l) be a complete D.L. space endowed with a graph K. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$, $S, T : M \to W(M)$ and let for a sequence $\{TS(c_n)\}$ in M generated by c_0, with $(c_0, c_1) \in F(K)$. Suppose that the following satisfy:*

(i) *S and T are graph dominated for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$;*
(ii) *there exists $\psi \in \Psi$ and*

$$D_l(c, g) = \max\left\{\begin{array}{c} d_l(c, g), \dfrac{d_l\left(c, [Tg]_{\beta(g)}\right) + d_l\left(g, [Sc]_{\gamma(c)}\right)}{2}, \\ \dfrac{d_l\left(c, [Sc]_{\gamma(c)}\right).d_l\left(g, [Tg]_{\beta(g)}\right)}{a + d_l(c, g)}, d_l(c, [Sc]_{\gamma(c)}), \\ d_l(g, [Tg]_{\beta(g)}) \end{array}\right\},$$

where $a > 0$, such that

$$H_{d_l}([Sc]_{\gamma(c)}, [Tg]_{\beta(g)}) \leq \psi(D_l(c, g)), \quad (3.1)$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$, and $(c, g) \in F(K)$ or $(g, c) \in F(K)$;

(iii) $\sum_{i=0}^{n} \psi^i(d_l(c_0, [Sc_0]_{\gamma(c_0)})) \leq r$ *for all* $n \in N \cup \{0\}$.

Then, $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$, $(c_n, c_{n+1}) \in F(K)$ as the sequence $\{TS(c_n)\} \to c^*$. Also, if $(c_n, c^*) \in F(K)$ or $(c^*, c_n) \in F(K)$ for all $n \in N \cup \{0\}$ and the inequality (3.1) holds for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\} \cup \{c^*\}$.Then S and T have common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0, r)}$.

Proof. Define, $\alpha : M \times M \to [0, \infty)$ by

$$\alpha(c, g) = \begin{cases} 1, & \text{if } c, g \in F(K) \\ 0, & \text{otherwise.} \end{cases}$$

As $\{TS(c_n)\}$ is a sequence in c generated by c_0 with $(c_0, c_1) \in F(K)$, we have $\alpha(c_0, c_1) \geq 1$. Let, $\alpha(c, g) \geq 1$, then $(c, g) \in F(K)$. From (i) we have $(c, [Sc]_{\gamma(c)}) \in F(K)$ for all $g \in [Sc]_{\gamma(c)}$ this implies that $\alpha(c, g) = 1$ for all $g \in [Sc]_{\gamma(c)}$. This further implies that $\inf\{\alpha(c, g) : g \in [Sc]_{\gamma(c)}\} = 1$. Thus S is a α_*-dominated multifunction on $\overline{B_{d_l}(c_0, r)}$. Also if $(c, g) \in F(K)$, we have $\alpha(c, g) = 1$ and hence $\alpha_*(c, [Sc]_{\gamma(c)}) = 1$. Similarly it can be proved $\alpha_*(g, [Tg]_{\beta(g)}) = 1$. Now,condition (ii) can be written as

$$H_{d_l}([Sc]_{\gamma(c)}, [Tg]_{\beta(g)}) \leq \psi(D_l(c, g)),$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\}$ with either $\alpha(c, g) \geq 1$ or $\alpha(g, c) \geq 1$. By including condition (iii), we obtain all the conditions of Theorem 2.1. Now, by Theorem 2.1, we have $\{TS(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \geq 1$, that is $(c_n, c_{n+1}) \in F(K)$ and $\{TS(c_n)\} \to c^* \in \overline{B_{d_l}(c_0, r)}$.Also, if $(c_n, c^*) \in F(K)$ or $(c^*, c_n) \in F(K)$ for all $n \in N \cup \{0\}$ and the inequality (3.1) holds for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\} \cup \{c^*\}$. Then, we have $\alpha(c_n, c^*) \geq 1$ or $\alpha(c^*, c_n) \geq 1$ for all $n \in N \cup \{0\}$ and the inequality (2.1) holds for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{TS(c_n)\} \cup \{c^*\}$. Again, by Theorem 2.1, S and T have common fuzzy fixed point c^* in $\overline{B_{d_l}(c_0, r)}$.

4. Fixed point results for multi-valued mapping

In this section, we show that Theorem 2.1 can be utilized to derive a common fixed point for a multi-valued mapping in a dislocated metric space.

Theorem 4.1. *Let (M, d_l) be a complete D.L. space. Suppose there exist a function $\alpha : M \times M \to [0, \infty)$. Let, $r > 0$, $c_0 \in \overline{B_{d_l}(c_0, r)}$ and $G, H : M \to P(M)$ be a α_*-dominated mappings on $\overline{B_{d_l}(c_0, r)}$. Assume that, for some $\psi \in \Psi$ and*

$$D_{l'''}(c, g) = \max \left\{ \begin{array}{c} d_l(c, g), \dfrac{d_l(c, Hg) + d_l(g, Gc)}{2}, \dfrac{d_l(c, Gc).d_l(g, Hg)}{a + d_l(c, g)}, \\ d_l(c, Gc), d_l(g, Hg) \end{array} \right\},$$

where $a > 0$ the following hold:

$$H_{d_l}(Gc, Hg) \leq \psi(D_{l'''}(c, g)), \quad (4.1)$$

for all $c, g \in \overline{B_{d_l}(c_0, r)} \cap \{HG(c_n)\}$, with either $\alpha(c, g) \geq 1$ or $\alpha(g, c) \geq 1$. Also

$$\sum_{i=0}^{n} \psi^i(d_l(c_0, Gc_0)) \leq r \text{ for all } n \in N \cup \{0\}. \quad (4.2)$$

Then $\{HG(c_n)\}$ is a sequence in $\overline{B_{d_l}(c_0, r)}$, $\alpha(c_n, c_{n+1}) \geq 1$ for all $n \in N \cup \{0\}$ and $\{HG(c_n)\} \to c^* \in \overline{B_{d_l}(c_0, r)}$. Also if $\alpha(c_n, c^*) \geq 1$ or $\alpha(c^*, c_n) \geq 1$ for all $n \in N \cup \{0\}$ and the inequality (2.1) holds for c^* also. Then G and H have common fixed point c^* in $\overline{B_{d_l}(c_0, r)}$.

Proof. Let $\delta : M \to (0, 1]$ be an arbitrary mapping. Consider $S, T : M \to W(M)$ be the two fuzzy mappings defined as

$$(Sc)(t) = \begin{cases} \delta(c) & \text{if } t \in Gc \\ 0 & \text{if } t \notin Gc \end{cases}$$

and

$$(Tc)(t) = \begin{cases} \delta(c) & \text{if } t \in Hc \\ 0 & \text{if } t \notin Hc \end{cases}.$$

We get

$$[Sc]_{\gamma(c)} = \{t : (Sc)(t) \geq \gamma(c)\} = Gc$$

and

$$[Tc]_{\beta(c)} = \{t : (Tc)(t) \geq \beta(c)\} = Hc.$$

So, the conditions (4.1) and (4.2) of Theorem 4.1 becomes the conditions (2.1) and (2.2) of Theorem 2.1. This implies that there exists $c^* \in [Sc]_{\gamma(c)} \cap [Tc]_{\beta(c)} = Gc \cap Hc$.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

Each author equally contributed to this paper, read and approved the final manuscript.

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